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This paper must be cited as:

Alfa, AS.; Pla, V.; Martínez Bauset, J.; Casares Giner, V. (2016). Discrete time analysis of cognitive radio networks with imperfect sensing and saturated source of secondary users, Computer Communications. Computer Communications. 79:53-65. doi:10.1016/j.comcom.2015.11.012.

The final publication is available at

http://dx.doi.org/10.1016/j.comcom.2015.11.012

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Additional Information

Discrete time analysis of cognitive radio networks with imperfect sensing and saturated source of secondary users

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Abstract

Sensing is one of the most challenging issues in cognitive radio networks. Selection of sensing parameters raises several tradeoffs between spectral efficiency, energy efficiency and interference caused to primary users (PUs). In this paper we provide representative mathematical models that can be used to analyze sensing strategies under a wide range of conditions. The activity of PUs in a licensed channel is modeled as a sequence of busy and idle periods, which are represented as alternating Markov phase renewal processes. The representation of the secondary users (SUs) behavior is also largely general: the duration of transmissions, sensing periods and the intervals between consecutive sensing periods are modeled by *phase type* distributions, which constitute a very versatile class of distributions. Expressions for several key performance measures in cognitive radio networks are obtained from the analysis of the model. Most notably, we derive the distribution of the length of an *effective white space*; the distributions of the waiting times until the SU transmits a given amount of data, through several transmission epochs or uninterruptedly; and the *goodput* when an interrupted SU transmission has to be restarted from the beginning due to the presence of a PU.

1. Introduction

Cognitive radio (CR) has been proposed as one approach to deal with the limited unlicensed spectrum availability [1]. Most of the licensed spectrum bands are not fully utilized most of the time. Capturing the unused time slots, which are known as spectrum holes, on these licensed spectrum bands and using them efficiently is what CR is about.

Depending on how the licensed spectrum is accessed, the operation of CR can be classified into two categories: *overlay* and *underlay*. In overlay CR, a licensed channel can be accessed by secondary users (SUs) only if it is not being used by a primary user (PU). Consequently, SUs can only transmit during the idle periods of the primary network. In contrast, in underlay CR, given that PU signals may be successfully decoded if the interference generated by the other sources is tolerable, SUs can access the spectrum even when there is a PU transmitting, as long as the

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generated interference at the PU receiver is lower than a pre-defined threshold. More recently, hybrid approaches combining overlay and underlay paradigms have been proposed [2]. In this paper we focus on overlay CR.

The key to success for CR consists of effective and efficient sensing of the channels by the SUs, the unlicensed users, to detect *white spaces* when they occur. A decision making process then follows the sensing, based on the outcome of the sensing exercise.

If we observe the activity of PUs in a licensed channel —that is, a certain spectrum band in a certain spatial area—, it is either busy or idle. For that channel and in that location, the idle periods are the spectrum holes referred to as *white space*. The busy and idle periods form an alternating sequence of events which may be correlated. An SU sensing this channel at some time points either finds the channel busy or idle. When SUs sense the channel during a *white space* then the SU is able to effectively utilize the remaining *white space*, which we call the *effective white space*. The ideal sensing strategy is one that results in the SU capturing most of the *white space* and ceasing its transmission shortly after a busy periods starts, i.e. the one that can minimize the difference between the lengths of the *white space* and the *effective white space*, while keeping resource utilization (e.g., energy) and interference to a minimum. Here, by sensing strategies we imply using different statistical distributions and mean values for the durations of the sensing periods and of the intervals between sensing periods.

As pointed out by Yücek and Arslan [3], sensing is one of the most important and challenging issues in CR networks. Selection of sensing parameters raises several tradeoffs involving achievable throughput, energy consumption and interference caused to PUs [4–11]. In general, more frequent and longer sensing periods reduce the achievable throughput, but increase the reliability of sensing and lower the interference that is caused to PUs. This is the so-called *sensing-throughput tradeoff* (STT) that has been addressed in many studies; see, for example, [4, 5, 9] and references therein. In the majority of these studies, achievable throughput is used as the main performance metric. To obtain the achievable throughput it is assumed that SUs always have data to transmit and that there is no overhead due to the coordination and/or collisions between SUs; in this paper we follow the same approach. In that sense, the achievable throughput is a useful measure to assess how well a sensing strategy does in the STT, but it must be made clear that throughput figures so obtained are not really achievable. In real settings, coordination among SUs in the same network and coexistence mechanisms among different secondary networks will unavoidably reduce the maximum throughput that SUs can obtain [12].

Sensing more often and doing it for longer periods increases energy consumption, which can be a critical aspect in some applications (e.g., sensor networks). This has brought energy consumption into the equation and some studies treated energy efficiency as a fundamental part of spectrum sensing [7, 8, 10, 11]. As a matter of fact, energy efficiency has become a major issue in communication networks, not only for battery operated devices but for all type of devices and infrastructures [13].

Scanning a wide frequency band in search of unused channels and switching channels (when a PU appears in the currently used one) are power-intensive CR operations [14]. Therefore, in those scenarios where energy consumption is critical, a common strategy for SUs upon detecting the presence of a PU is to remain silent in the current channel and wait for the channel to become idle again, instead of performing a spectrum handoff to switch to a different channel [8, 15]. In this paper we focus on this type of strategies.

Most of the existing works that aim to optimize spectrum sensing parameters assume that PUs transmissions have a temporal structure that is known by the CR network. The CR network can then structure its transmission in frames that are temporarily aligned with those of PUs. This way, the CR network can sense the state of the channel at the beginning of the frame, and rest assured that whatever the outcome of the sensing was it will not change throughout the remaining of the frame. This assumption has been relaxed in recent papers in which it is considered that a PU can change its status at anytime during an SU frame [6, 16, 17]. Here we follow the latter more general approach.

Another aspect which is often overlooked in the studies dealing with spectrum sensing is the effect that collisions with PUs may have on the effective throughput of SUs [5, 6]. Collisions with PUs may occur due to either sensing errors or the sudden arrival of a PU while the SU is transmitting. As noted in [5], this effect and its impact on the effective throughput has been rarely addressed in the literature to date. In this paper the impact of PU interference on SU effective throughput is considered in the analysis of the *goodput* as explained below.

The vast majority of models that have been developed for studying CR networks, and in particular for studying spectrum sensing, assume that the durations of both the busy and idle times are governed by the exponential distribution (or its discrete time counterpart: geometric), see for example [18, 19] and all the references given above for sensing related studies. However, this assumption is not backed with empirical evidence. Geirhofer *et al.* [20] were among the earlier researchers to show from measurements that the idle time duration was more of a lognormal distribution. This was further confirmed by Wellens *et al.* [21] who showed that the idle time duration has lognormal distribution for long durations and geometric distribution for the short ones. Most of the previous works also ignored the possible correlation between the busy and idle times, even though logically one would expect some correlation to exist between the two intervals, and between consecutive busy and idle periods as observed in [21].

The study of sensing strategies in CR networks is not a new topic, nor is the development of mathematical models for their analysis and optimization. However, the models presented in this paper contain several contributions arising from the generality of the assumptions and the performance measures derived. Our aim is to provide representative mathematical models that can be used to analyze sensing strategies under a wide range of conditions. We do this by representing the busy and idle periods of PUs in a licensed channel as alternating Markov phase renewal processes [22] and develop a model to determine relevant performance measures under different sensing strategies. This Markov phase renewal process allows the busy and idle times durations to assume a wide variety of distributions and also captures much broader correlation aspects of the two intervals. Our model of PUs activity is similar to the one by Bae *et al.* in [23], although their model did not consider the correlations between busy and idle periods.

Our model of the SU behavior is also largely general. The duration of transmissions, sensing periods, and intervals between consecutive sensing periods are modeled by general *phase type* distributions [22]. In the modeling literature, it is acknowledged that the phase type distribution provides an excellent balance between tractability and applicability (it is general enough to fit empirical data and there exist algorithms for this purpose) [24]. Moreover, in the current paper we assume that sensing is imperfect and consider two types of misdetections and false alarms, depending on whether the SU is transmitting or not. This, allows us to cover a wide range of situations.

As mentioned above, in our modeling approach a number of random variables that refer to durations are described by discrete time phase type distributions. A known advantage of this type of distributions is that a constant value (i.e., a deterministic distribution) can be easily represented as a discrete time phase type distribution. It is worth noting this point since most of the proposed sensing schemes consider constant values for the sensing period and for the interval between sensing periods [4]. The often made assumption is that sensing should be done periodically and the issue is determining the interval between the sensing times [5, 25]. However, there are no documented studies supporting the idea of using constant interval between sensing times as the optimal strategy. Therefore, while our model can be easily set so that sensing related durations are constant, allowing these durations to be general does not involve much extra modeling effort, and makes the model more general.

An important part of the sensing strategy are the physical layer functions aimed at determining whether a certain spectrum band is being used or not. These functions rely on signal processing techniques and are usually known as *spectrum sensing algorithms*. In the literature, a number of spectrum sensing algorithms have been proposed (see [3, 26, 27] for an overview). Although it is an important part of the sensing strategy, the spectrum sensing algorithm itself is outside the scope of this paper and its detailed operation is not described by our model. In that regard, our model is not tied to any specific spectrum sensing algorithm. It can be setup to model sensing strategies based on different specific spectrum sensing algorithms by setting appropriately the misdetection and false alarm probabilities; the duration assigned to a time slot and the energy consumed per slot by sensing activities will also vary depending on the spectrum sensing algorithm. In the case of cooperative spectrum sensing algorithms, there exists an additional delay until the decision is made due to the required reporting and fusion of the distributed measurements [28]. Our model does not include this delay overhead. As a consequence, it could only be used with cooperative sensing schemes whose delay overhead can be considered negligible.

From the analysis of our model we obtain the expressions for several key performance measures in CR networks. Most notably, we derive the distribution of the length of an *effective white space*, and the distributions of the waiting times until the SU transmits a given amount of data in total (through possibly several transmissions separated by interruptions or uninterruptedly). We also obtain an expression for the *goodput* when an interrupted SU transmission has to be started over from the beginning (non-resume transmission). The correctness of the analysis of the model and its numerical implementation has been validated through computer simulations.

The main contributions of the paper can be summarized as follows. First, we consider a model of the PUs activity that is largely general and can be fitted to mimic the empirically observed behaviors. To the best of our knowledge it is the most general model of this kind that has been proposed so far. Second, the model of the SU behavior is also largely general. Specifically, we are not aware of any previous mathematical modeling study in which general distributions are used for the duration of transmissions, sensing periods, and intervals between consecutive sensing periods. And third, in addition to the common performance metrics based on mean values (e.g. throughput and interference caused to

PUs) we derive expressions for their distribution and other additional key performance measures, namely, the waiting time distributions for transmitting a given amount of data under two different criteria, and the goodput (effective throughput).

The rest of the paper is organized as follows. Section 2 introduces the model of the channel as seen by SUs, i.e., the pattern of channel busy periods and white spaces. The behavior of the CR system is described in Section 3. Then, a Markov chain model of the system is developed and some performance measures, including the distributions of two different waiting times, are derived in Section 4. In Section 5 we present a number of numerical studies looking at several aspects of the performance of the CR network to illustrate the versatility of the developed model. Finally, the paper is concluded in Section 6.

2. Characteristics of the channel

We observe our system in discrete time points $0, 1, 2, \ldots$. Consider a single licensed communication channel used by PUs. This channel is either in use (*busy*) or not in use (*idle*). Throughout the paper the channel condition (*busy* or *idle*) only concerns the PUs activity in it: if there is no PU using the channel it is considered to be *idle*, even if there was an SU transmitting on this channel. Hence this is viewed from the perspective of the PUs.

Consider a discrete time Markov chain (DTMC) with state space $\{1, 2, \cdots, n_b, n_b + 1, \cdots, n_b + n_i\}$, with the transition matrix representing this DTMC given as D. Let the channel *busy* (b) and *idle* (i) conditions be represented by the subsets of states $\{1, 2, \cdots, n_b\}$ and $\{n_b + 1, \cdots, n_b + n_i\}$, respectively. In other words, the channel is *busy* whenever the DTMC is in any of the n_b states $\{1, 2, \cdots, n_b\}$, and it is *idle* when the DTMC in any of the n_i states $\{n_b + 1, \cdots, n_b + n_i\}$. Further define the substochastic matrices D_b and D_i to represent the transitions within the busy and idle states, respectively, d_{bi} to represent transitions from busy to idle states, and d_{ib} from idle to busy states. The substochastic matrices D_b and D_i are of orders n_b and n_i, respectively. By these definitions we have $D_b\mathbf{1} + d_{bi}\mathbf{1} = \mathbf{1}$ and $D_i\mathbf{1} + d_{ib}\mathbf{1} = \mathbf{1}$, where $\mathbf{1}$ is a column vector of ones of appropriate dimensions [1]. Based on this, the channel is either busy or idle according to the following transition matrix

$$D = \begin{bmatrix} D_b & d_{bi} \\ d_{ib} & D_i \end{bmatrix}. \tag{1}$$

Let $\boldsymbol{\pi} = [\boldsymbol{\pi}_b, \boldsymbol{\pi}_i]$ be the stationary distribution, where $\boldsymbol{\pi} = \boldsymbol{\pi} D$ and $\boldsymbol{\pi}\mathbf{1} = 1$. This is a realistic representation of the channel behavior under general conditions.

The behavior of the channel (regarding PU activity) can be seen as an alternating sequence of busy and idle periods (white spaces). Let Δ represent the duration of a white space, which follows a phase type distribution [22] with representation $((\boldsymbol{\pi}_b d_{bi}\mathbf{1})^{-1}\boldsymbol{\pi}_b d_{bi}, D_i)$.

Therefore, the probability that the duration of uninterrupted white space is at least k time slots, is given as

$$P(\Delta \geq k) = \frac{1}{\boldsymbol{\pi}_b d_{bi}\mathbf{1}}\boldsymbol{\pi}_b d_{bi} D_i^{k-1}\mathbf{1}, \quad k \geq 1. \tag{2}$$

[1] A summary of the notation introduced in the paper is given in Table 1

Table 1: Summary of notation

General	
Column vector of ones of appropriate dimensions	$\mathbf{1}$
Channel activity (PU)	
Number of busy (idle) states	n_b (n_i)
Transition matrix from busy (idle) states to busy (idle) states	D_b (D_i)
Transition matrix from busy (idle) states to idle (busy) states	d_{bi} (d_{ib})
Duration of a white space	Δ
Utilization factor of the channel (by PUs)	ρ_{PU}
Average burst length	$\mathrm{E}[B]$
SU	
Sleep length; phase type representation	$(\boldsymbol{\delta}, L)$, $\boldsymbol{l} = \mathbf{1} - L\mathbf{1}$
Sensing length; phase type representation	$(\boldsymbol{\beta}, S)$, $\boldsymbol{s} = \mathbf{1} - S\mathbf{1}$
Transmission length; phase type representation	$(\boldsymbol{\alpha}, T)$, $\boldsymbol{t} = \mathbf{1} - T\mathbf{1}$
Slot misdetection probability in a slot when not transmitting	ϕ_1
Slot false alarm probability in a slot when not transmitting	θ_1
Slot misdetection probability in a slot when not transmitting	ϕ_2
Slot false alarm probability in a slot when not transmitting	θ_2
Performance parameters	
Duration of an effective white space	Δ^e
Number of time slots in an effective white space that correspond to a successfully completed transmission	Δ^g
Efficiency factors	η_s, η_t, η_g $\eta = \alpha\eta_t + (1-\alpha)\eta_s$
Duration of an interference run	Ψ
Fraction of PU transmission slots interfered by the SU	I_f
Goodput (effective throughput)	γ
Waiting time to obtain m slots of effective white space	W_m
Waiting time before obtaining m consecutive slots of effective white space	W_m^c

In the equation above, the entries of the vector $(\boldsymbol{\pi}_b d_{bi}\mathbf{1})^{-1}\boldsymbol{\pi}_b d_{bi}$ are the probabilities of the idle period being initiated at each of the n_i states, $\{n_b+1, \cdots, n_b+n_i\}$. Observe that these initiation probabilities correspond to the probabilities of landing at each of the n_i states in the idle condition of the channel, conditioned on the fact that a transition from *busy* to *idle* has occurred. The element $(D_i^{k-1})_{\ell,m}$ is the probability of starting from state $n_b+\ell$ and ending up in state n_b+m after $k-1$ time slots without leaving the set of idle states $\{n_b+1, \cdots, n_b+n_i\}$.

From (2) it readily follows that the average duration of a white space is

$$\mathrm{E}[\Delta] = \sum_{k=1}^{\infty} P(\Delta \geq k) = \frac{1}{\boldsymbol{\pi}_b d_{bi}\mathbf{1}}\boldsymbol{\pi}_b d_{bi}\left(\sum_{k=1}^{\infty} D_i^{k-1}\right)\mathbf{1} = \frac{\boldsymbol{\pi}_b d_{bi}(I-D_i)^{-1}\mathbf{1}}{\boldsymbol{\pi}_b d_{bi}\mathbf{1}} = \frac{\boldsymbol{\pi}_i\mathbf{1}}{\boldsymbol{\pi}_i(I-D_i)\mathbf{1}}, \tag{3}$$

where the last equality follows from the fact that $\boldsymbol{\pi} D = \boldsymbol{\pi}$, which implies that $\boldsymbol{\pi}_b d_{bi} = \boldsymbol{\pi}_i(I-D_i)$.

As an example, we next construct the matrix D for a case in which the PUs arrive randomly, occupy the channel if there is no other PU using it, or are lost otherwise. In this model, both the arrival process and channel holding time for PUs are rather general.

Example of Channel Model. Consider a single channel used by PUs. Let the PUs arrive according to the Markovian arrival process (MAP) represented by two matrices A_0 and A_1 of dimension n_a.

The MAP is considered a good model to capture packet arrival processes in modern communication networks. Using a MAP we can fit a wide variety of packet interarrival time distributions and autocorrelations [29]. For broadband wired links, which usually transport traffic with a certain degree of aggregation, the batch extension of MAP rather than MAP itself is normally proposed as the appropriate model [30]. Nonetheless, for wireless links or when traffic comes from a single source, the MAP has proven to be sufficiently general [31, 32]. Therefore, in our case we can safely restrict ourselves to the MAP.

The MAP can be defined by considering an $(n_a + 1)$ state absorbing Markov chain with state space $\{0, 1, 2, \ldots, n_a\}$ in which state 0 is the absorbing state. Then, A_0 and A_1 are two sub-stochastic matrices such that: the element $(A_0)_{k,l}$ refers to a transition from state k to state l without an arrival; the element $(A_1)_{k,l}$ refers to a transition from state k into the absorbing state 0, with an instantaneous restart from the transient state l and with an arrival during the absorption. The matrix $A = A_0 + A_1$ is a stochastic matrix, and we assume it is irreducible. The interested reader is referred to [22, Sec. 3.3.5] for further details about MAPs. We note that the phase from which an absorption occurred and the one from which the next process starts are connected and hence this captures the correlation between interarrival times. This makes the MAP widely used for non-renewal type arrival processes in the performance evaluation of telecommunication networks, since it can capture the properties of both burstiness and correlations.

Any PU that arrives to find the channel in use by another PU is assumed to be lost. Suppose now that when a PU arrives it finds the channel idle. In that case, it is able to connect and then hold the channel for a random length of time which has a discrete time phase type distribution with representation $(\boldsymbol{\omega}, G)$ of dimension n_g. A phase type distribution represents the time to absorption in an absorbing Markov chain. For a phase type distribution of dimension n_g with representation $(\boldsymbol{\omega}, G)$, there are n_g transient states and one absorbing state. The element ω_i of the probability vector $\boldsymbol{\omega} = [\omega_1, \ \ldots, \ \omega_{n_g}]$ is the probability of initially starting from the state i. The substochastic matrix G contains the transition probabilities between transient states, and the elements of the column vector $\boldsymbol{g} = \mathbf{1} - G\mathbf{1}$ represent the transition probabilities between the transient states and the absorbing state. The interested reader is referred to [22, Secs. 2.6 and 3.3.2] for further details about phase type distributions.

Then, from the perspective of PUs, the channel is either busy with one PU or it is idle (i.e., without a PU using it). Essentially if the channel is in use by an SU then it is, for all intents and purposes, idle as assessed by a PU. Consider the state space $\{\mathbf{b}, \mathbf{i}\}$, where $\mathbf{b} = \{1, 2, \cdots, W\}$ and $\mathbf{i} = \{W+1, W+2, \cdots, W+n_a\}$, and $W = n_g \cdot n_a$. Then the transition matrix P_c representing the DTMC associated with the states of the channel can be written as

$$P_c = \begin{bmatrix} A \otimes G + A_1 \otimes (\boldsymbol{g}\boldsymbol{\omega}) & A_0 \otimes \boldsymbol{g} \\ A_1 \otimes \boldsymbol{\omega} & A_0 \end{bmatrix}, \tag{4}$$

where the symbol $\otimes$ denotes the Kronecker product [22, page 30]. The blocks of $D = P_c$ can be explained as follows. In the discussion below we deal with probabilities in *matrix form*. We refer to the matrix blocks as probability matrices of going from one state (which is composed of several phases) to another state (which is also composed of several phases), so that the row and the column of the block matrix represent the phase in the initial and final state, respectively. The upper left block $D_b = A \otimes G + A_1 \otimes (\boldsymbol{g}\boldsymbol{\omega}) = A_0 \otimes G + A_1 \otimes (\boldsymbol{g}\boldsymbol{\omega} + G)$, is the sum of the probabilities that: no arrival occurs and the ongoing service does not terminate ($A_0 \otimes G$); an arrival occurs, the ongoing services terminates and the service of the newly arrived customer begins ($A_1 \otimes (\boldsymbol{g}\boldsymbol{\omega})$); and an arrival occurs but the ongoing service keeps on, i.e. the arriving customer is blocked, ($A_1 \otimes G$). The upper right block $d_{bi} = A_0 \otimes \boldsymbol{g}$ represents the probability of no arrival occurring and the termination of the ongoing service. The lower left block $d_{ib} = A_1 \otimes \boldsymbol{\omega}$ represents the probability of an arrival and the beginning of its service. Finally, the lower right block $D_i = A_0$ represents the probability of no arrival.

3. Model of the Cognitive Radio System

In this section we consider the actions of the SUs and how they interact with the PUs. Throughout the paper, by SU we refer to the aggregate of all SUs which might use the channel under consideration, between which there exists some sort of coordination. The SU can be in one of the following three situations: *sleeping*, *sensing* or *transmitting*.

Sleeping: The duration of the sleeping state is represented by the phase type distribution $(\boldsymbol{\delta}, L)$. After waking up, the SU proceeds to the sensing period. *Sleeping* simply means the SU is not active, and thus its energy consumption is reduced to a minimum. Ideally, the SU should be sleeping as long the channel is being used by PUs. However, in most cases it is difficult to precisely anticipate when the channel will become idle. If the duration of the sleep phase is extended so that the SU remains sleeping for too long after the channel became idle, the duration (and even the number) of effective white spaces will decrease. In contrast, if the duration of the sleep phase is reduced and the SU wakes up too many times during a channel busy period, its energy consumption will increase unnecessarily.

Sensing: In the sensing period the SU takes a series of measurements of the channel state in consecutive time slots. The maximum number of such measurements in a sensing period is described by the phase type distribution $(\boldsymbol{\beta}, S)$. If the outcome of any of the measurements is *busy* the sensing period is aborted and the SU goes back to sleep. Otherwise, if all measurements in the series find the channel idle, the SU proceeds to transmit.

Transmitting: The SU will transmit for a maximum number of slots represented by the phase type distribution $(\boldsymbol{\alpha}, T)$. These number of slots corresponds to the length of a message. Depending on the sensing characteristics and capabilities of the SU, it may, or may not, be able to detect the arrival of a PU during the transmission period. After successfully finishing a transmission the SU enters a sensing period. When sensing is carried out during transmission, if the presence of a PU is sensed then the SU interrupts the transmission and enters a sleeping cycle.

Let the source of SUs be *saturated*, i.e. there is always an SU looking for white space. We also assume that the system gives preemptive priority to PUs over the SUs. In a separate paper, we will consider the case where we allow the system to keep track of the phase at which the SU was interrupted, and also relax the assumption about the source of SUs being saturated.

The duration of a slot represents the length of time required to obtain a sensing measurement. The outcome of the sensing measurement performed during a slot is known at the end of the slot. In setting the result of the measurement we adopt a conservative approach: if any PU activity occurred during the slot, the channel state will be deemed as busy. In other words, the measurement undertaken during the time slot $[n, n+1)$ will yield *idle* only if the channel was idle at both $t = n$ and $t = n + 1$.

In a previous paper [33] we had assumed perfect sensing. Here, in contrast, we consider that sensing is imperfect and consider two types of misdetections and false alarms. Type 1 misdetection occurs when the SU senses the channel and mistakenly assesses it as idle when it is busy. The probability of this happening is ϕ_1, with $\bar{\phi_1} = 1 - \phi_1$. Type 2 misdetection occurs when a PU arrives into the system while an SU is transmitting, and the SU does not detect the PU arrival and continues transmitting. The probability of this happening is ϕ_2, with $\bar{\phi_2} = 1 - \phi_2$. Finally, there are also false alarms. They can occur either when the SU senses the channel and assesses it to be busy when it is actually idle. This is false alarm type 1 and we assign it the probability θ_1, with $\bar{\theta}_1 = 1 - \theta_1$. Also when the SU is transmitting it may falsely sense the PU arrival when actually it is not true. The probability of this is θ_2 with $\bar{\theta}_2 = 1 - \theta_2$. These parameters allow us to cover a wide range of situations. For example, $\phi_2 = 1, \theta_2 = 0$ for the case where the SU cannot sense while it is transmitting; $\phi_1 = \phi_2 = 0, \theta_1 = \theta_2 = 0$ for the case of perfect sensing capability regardless the SU is transmitting or not; and $0 < \phi_1 < \phi_2 < 1, 0 < \theta_1 < \theta_2 < 1$ for a case of imperfect sensing that is less accurate while the SU is transmitting.

Despite the fact that the sensing time cannot be neglected, it is assumed that sensing may be carried out while the SU is transmitting. This would correspond to those situations in which the SU is equipped with the required hardware to transmit and sense simultaneously (dual radio architecture [3]), or in which the sensing is carried out by a different SU device and the sensing outcome is communicated to the transmitting SU at the end of the slot over a dedicated control channel [34]. Note that, as illustrated by the examples in the previous paragraph, our model covers the two possibilities: sensing and transmitting can and cannot be carried out simultaneously. Furthermore, when sensing and transmitting can be carried out simultaneously, our model allows to consider different accuracies for the sensing performed when the SU is transmitting and when it is idle.

The probabilities of misdetection and false alarm introduced above refer to these events occurring at a time slot, i.e., to measurements taken during a single time slot. However, in the sensing strategy that we consider here there may be several consecutive sensing slots in a sensing period. Besides, even if all measurements taken during a sensing period yield a correct result, it may happen that the PU state changes just after the sensing period ended and, as a consequence, the SU starts to transmit when the PU is busy, or goes to sleep when the PU is idle.

In this paper we use the term *global misdetection* to refer to the situation in which after a sensing period the SU starts transmitting and collides with a PU at the start of the transmission. Analogously, we use the term *global false alarm* to refer to the situation in which after a sensing period the SU goes to sleep but the channel happens to be idle at the start of the sleep period.

In Section 4.4 we derive the probabilities of global misdetection and global false alarm. As might be expected, these probabilities depend not only in the per slot misdetection and false alarm probabilities, but also on the sensing strategy and the PU dynamics.

We now use a DTMC with the following states (again, each of these states is composed of several Markovian phases) to study the performance of this system

- State 1 = (channel busy, SU sleeping),
- State 2 = (channel busy, SU sensing),
- State 3 = (channel busy, SU transmitting),
- State 4 = (channel idle, SU sleeping),
- State 5 = (channel idle, SU sensing), and
- State 6 = (channel idle, SU transmitting).

The transition matrix of this DTMC can be written as

$$P = \begin{bmatrix} P_{11} & P_{12} & 0 & P_{14} & P_{15} & 0 \\ P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & P_{36} \\ P_{41} & P_{42} & 0 & P_{44} & P_{45} & 0 \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} & P_{56} \\ P_{61} & P_{62} & P_{63} & P_{64} & P_{65} & P_{66} \end{bmatrix}, \tag{5}$$

where the block elements P_{ij} are as defined in the detailed matrix P shown in the Appendix.

Provided the Markov chain described by the matrix P is irreducible then it has a stationary distribution $\boldsymbol{\pi}$ given as

$$\boldsymbol{\pi} = \boldsymbol{\pi} P, \quad \boldsymbol{\pi}\mathbf{1} = 1,$$

where $\boldsymbol{\pi} = [\boldsymbol{\pi}_1,\ \boldsymbol{\pi}_2,\ \boldsymbol{\pi}_3,\ \boldsymbol{\pi}_4,\ \boldsymbol{\pi}_5,\ \boldsymbol{\pi}_6]$.

4. Performance Measures

4.1. Duration of an effective white space

Our interest here is in state 6. We are interested in the probability that, when the SU is finally able to transmit, it has at least k uninterrupted units of time slots for transmission without interfering with the PU.

Let us further group the states as $\{1,2,3,4,5\}$ and $\{6\}$. Since this type of grouping will be used several times throughout the paper, for abbreviation we introduce the notation $\bar{j} = \{1,2,3,4,5,6\} \setminus \{j\}$ to refer to the whole state space after removing the group of states in j. Now we rewrite the matrix P by appropriately reordering rows and columns as

$$P = \begin{bmatrix} P_{\bar{6},\bar{6}} & P_{\bar{6},6} \\ P_{6,\bar{6}} & P_{6,6} \end{bmatrix}. \tag{6}$$

Thus, the duration of an effective white space, which we denote by Δ^e, corresponds with the sojourn in state 6. Then, the probability the available uninterrupted transmission time without interference is at least k time slots is

$$P(\Delta^e \geq k) = \frac{1}{\boldsymbol{\pi}_{\bar{6}} P_{\bar{6},6} \mathbf{1}} \boldsymbol{\pi}_{\bar{6}} P_{\bar{6},6} P_{6,6}^{k-1} \mathbf{1} = \frac{\pi_6 (I - P_{6,6}) P_{6,6}^{k-1} \mathbf{1}}{\pi_6 (I - P_{6,6}) \mathbf{1}}. \tag{7}$$

In the equation above, the entries of the vector $(\boldsymbol{\pi}_{\bar{6}} P_{\bar{6},6} \mathbf{1})^{-1} \boldsymbol{\pi}_{\bar{6}} P_{\bar{6},6}$ are the probabilities of landing at each phase of state 6 conditioned on the fact that a transition from $\bar{6}$ to 6 has occurred, and $(P_{6,6}^{k-1})_{i,j}$ is the probability of starting from phase i of state 6 and ending up in phase j (of state 6) $k-1$ time slots later without leaving state 6 in between. Then the last equality follows by recalling that $\boldsymbol{\pi} P = \boldsymbol{\pi}$.

From Eq. (7) it readily follows that the mean effective white space, $\mathrm{E}[\Delta^e]$, is given as

$$\mathrm{E}[\Delta^e] = \sum_{k=1}^{\infty} P(\Delta^e \geq k) = \frac{\pi_6 \mathbf{1}}{\pi_6 (I - P_{6,6}) \mathbf{1}}. \tag{8}$$

4.2. Efficiency of the sensing strategy.

We assess the efficiency of the sensing strategy from two different viewpoints:

- The effectiveness in using available white spaces for transmission. We represent this by η_t, which is given as

$$\eta_t = \frac{\boldsymbol{\pi}_6 \mathbf{1}}{\boldsymbol{\pi}_4 \mathbf{1} + \boldsymbol{\pi}_5 \mathbf{1} + \boldsymbol{\pi}_6 \mathbf{1}} = \frac{\boldsymbol{\pi}_6 \mathbf{1}}{\boldsymbol{\pi}_i \mathbf{1}}, \tag{9}$$

  and represents the fraction of slots available for SU transmission in which it effectively transmits.

- The effectiveness in saving unnecessary measurements by remaining in the *sleeping* state while the channel is busy. We represent this by η_s, which is given as

$$\eta_s = \frac{\boldsymbol{\pi}_1 \mathbf{1}}{\boldsymbol{\pi}_1 \mathbf{1} + \boldsymbol{\pi}_2 \mathbf{1} + \boldsymbol{\pi}_3 \mathbf{1}} = \frac{\boldsymbol{\pi}_1 \mathbf{1}}{\boldsymbol{\pi}_b \mathbf{1}}, \tag{10}$$

  and represents the fraction of slots the SU is in the sleeping state while the channel is busy. Saving as much measurements as possible, i.e. a high value of η_s, is a desirable characteristic of sensing strategies as it means saving energy or being able to use the radio to sense other channels.

Obviously, we have that $0 \leq \eta_t, \eta_s \leq 1$. It is clear that there is a tradeoff between these two efficiency measures. Short sleeping periods leads to high values of η_t and low values of η_s, and vice versa. We summarize both measures into an overall efficiency factor, η, as

$$\eta = \alpha \eta_t + (1 - \alpha) \eta_s \,, \tag{11}$$

where $0 \leq \alpha \leq 1$.

4.3. Amount of interference caused to PUs

We denote by Ψ the duration of an interference run (series of consecutive slots where the channel is busy and the SU is transmitting). It can be analyzed in a similar way to what was done for the duration of an effective white space in (7) and (8). The probability that the interference run is at least k slots long is

$$P(\Psi \geq k) = \frac{1}{\boldsymbol{\pi}_{\bar{3}} P_{\bar{3},3} \mathbf{1}} \boldsymbol{\pi}_{\bar{3}} P_{\bar{3},3} P_{3,3}^{k-1} \mathbf{1} = \frac{\boldsymbol{\pi}_3 (I - P_{3,3}) P_{3,3}^{k-1} \mathbf{1}}{\boldsymbol{\pi}_3 (I - P_{3,3}) \mathbf{1}}, \tag{12}$$

and its mean duration is given by

$$\mathrm{E}[\Psi] = \sum_{k=1}^{\infty} P(\Psi \geq k) = \frac{\boldsymbol{\pi}_3 \mathbf{1}}{\boldsymbol{\pi}_3 (I - P_{3,3}) \mathbf{1}}. \tag{13}$$

Now, the fraction of slots in which a PU transmission is interfered by a simultaneous SU transmission is

$$I_f = \frac{\pi_3 \mathbf{1}}{\pi_1 \mathbf{1} + \pi_2 \mathbf{1} + \pi_3 \mathbf{1}} = \frac{\pi_3 \mathbf{1}}{\pi_b \mathbf{1}}. \tag{14}$$

4.4. Probabilities of global misdetection and global false alarm

A global misdetection occurs when, after a sensing period, the SU starts transmitting and collides with a PU at the start of the transmission. The probability of this happening is

$$P_{\mathrm{md}} = \frac{\boldsymbol{\pi}_2 P_{23} \mathbf{1} + \boldsymbol{\pi}_5 P_{53} \mathbf{1}}{\boldsymbol{\pi}_2 (P_{21} \mathbf{1} + P_{23} \mathbf{1} + P_{24} \mathbf{1} + P_{26} \mathbf{1}) + \boldsymbol{\pi}_5 (P_{51} \mathbf{1} + P_{53} \mathbf{1} + P_{54} \mathbf{1} + P_{56} \mathbf{1})}. \tag{15}$$

A global false alarm occurs when, after a sensing period, the SU goes to sleep but the channel happens to be idle at the start of the sleep period. The probability of this happening is

$$P_{\mathrm{fa}} = \frac{\boldsymbol{\pi}_2 P_{24} \mathbf{1} + \boldsymbol{\pi}_5 P_{54} \mathbf{1}}{\boldsymbol{\pi}_2 (P_{21} \mathbf{1} + P_{23} \mathbf{1} + P_{24} \mathbf{1} + P_{26} \mathbf{1}) + \boldsymbol{\pi}_5 (P_{51} \mathbf{1} + P_{53} \mathbf{1} + P_{54} \mathbf{1} + P_{56} \mathbf{1})}. \tag{16}$$

In Section 5.3 we study how the duration of the sensing period affects the probabilities of global misdetection and global false alarm.

4.5. SU goodput

Goodput, or effective throughput, is defined as the average number of effectively transmitted data units per time slot; where a data unit might correspond, for example, to the number of bits that can be transmitted in a time slot. In the throughput we count all time slots in which the SU transmits and the PU is idle. However, in some applications, messages need to be transmitted successfully (without collision with the PU) and uninterruptedly; otherwise the transmission of the message needs to restart from the beginning and all transmission slots for this message are wasted. Thus, in the goodput we only count the transmission slots of messages that are successfully transmitted in full.

There are several reasons that may prevent a message from being successfully transmitted without interruption: the SU starts to transmit while the PU is busy, owing to a misdetection; the PU becomes busy during the transmission of an SU message; or the transmission of a message is interrupted because of a (type 2) false alarm.

For a transmission to be successful, when the SU finishes the sensing period and proceeds to transmit the channel must be idle and remain in this state until the transmission is completely transmitted. In terms of states this means that the DTMC must enter state 6 from states 2 or 5, remain in state 6 for as long as required, and then leave state 6 to go to states 2 or 5. From this it follows that the mean number of successfully completed transmissions lasting exactly $k \geq 1$ time slots that are initiated per time slot is given by

$$\lambda_{s,k} = (\boldsymbol{\pi}_2 P_{2,6} + \boldsymbol{\pi}_2 P_{5,6}) P_{6,6}^{k-1} (P_{6,2}\mathbf{1} + P_{6,5}\mathbf{1}). \tag{17}$$

Therefore, the mean number of successfully completed transmissions that are initiated per time slot is

$$\lambda_s = \sum_{k=1}^{\infty} \lambda_{s,k} = (\boldsymbol{\pi}_2 P_{2,6} + \boldsymbol{\pi}_2 P_{5,6})(I - P_{6,6})^{-1}(P_{6,2}\mathbf{1} + P_{6,5}\mathbf{1}). \tag{18}$$

Similarly, the mean number of transmission runs when the channel is idle (i.e., effective white spaces or sojourns in state 6) that are initiated per time slot is

$$\lambda_t = \boldsymbol{\pi}_{\bar{6}} P_{\bar{6},6}\mathbf{1} = \boldsymbol{\pi}_6 (I - P_{6,6})\mathbf{1}. \tag{19}$$

Let us denote by Δ^g the number of time slots during an effective white space that correspond to successfully completed transmissions. We now write the expression for the distribution of Δ^g as

$$\mathrm{P}(\Delta^g = k) = \begin{cases} 1 - \dfrac{\lambda_s}{\lambda_t} & \text{if } k = 0, \\ \dfrac{\lambda_{s,k}}{\lambda_t} & \text{if } k = 1, 2, \ldots. \end{cases} \tag{20}$$

Then,

$$\mathrm{E}[\Delta^g] = \sum_{k=1}^{\infty} k\, \mathrm{P}(\Delta^g = k) = \frac{(\boldsymbol{\pi}_2 P_{2,6} + \boldsymbol{\pi}_2 P_{5,6})(I - P_{6,6})^{-2}(P_{6,2}\mathbf{1} + P_{6,5}\mathbf{1})}{\lambda_t}. \tag{21}$$

Now, we can write the fraction of time slots in an effective white space that correspond to successfully completed transmissions as

$$\frac{\mathrm{E}[\Delta^g]}{\mathrm{E}[\Delta^e]} = \frac{(\boldsymbol{\pi}_2 P_{2,6} + \boldsymbol{\pi}_2 P_{5,6})(I - P_{6,6})^{-2}(P_{6,2}\mathbf{1} + P_{6,5}\mathbf{1})}{\boldsymbol{\pi}_6 \mathbf{1}}, \tag{22}$$

where we have substituted the expression of $\mathrm{E}[\Delta^e]$ given in (8).

From this, the following expression for the goodput follows immediately

$$\gamma = \frac{\mathrm{E}[\Delta^g]}{\mathrm{E}[\Delta^e]} \boldsymbol{\pi}_6 \mathbf{1} = (\boldsymbol{\pi}_2 P_{2,6} + \boldsymbol{\pi}_2 P_{5,6})(I - P_{6,6})^{-2}(P_{6,2}\mathbf{1} + P_{6,5}\mathbf{1}). \tag{23}$$

Similarly to what is done in (9), we define

$$\eta_g = \frac{\gamma}{\boldsymbol{\pi}_4 \mathbf{1} + \boldsymbol{\pi}_5 \mathbf{1} + \boldsymbol{\pi}_6 \mathbf{1}} = \frac{(\boldsymbol{\pi}_2 P_{2,6} + \boldsymbol{\pi}_2 P_{5,6})(I - P_{6,6})^{-2}(P_{6,2}\mathbf{1} + P_{6,5}\mathbf{1})}{\boldsymbol{\pi}_i \mathbf{1}}, \tag{24}$$

which represents the fraction of time slots used for effective SU transmission when the SU has to transmit messages uninterruptedly and without colliding with the PU.

4.6. Waiting time for a fixed total length of white spaces

Of interest to us some times is how long it takes to get a particular total length of white spaces. For example, we may be seeking a total of 5 time slots of white spaces in order to transmit a particular message. If we wait through 3 inaccessible time slots and then get 2 time slots of white spaces and again wait 10 inaccessible time slots and get 3 time slots of white spaces, then it has taken us 18 units of time to get 5 units of white spaces in order to complete the message transmission.

Let $B = \{6\}$ represent the state corresponding to an effective white space (the channel is idle, and the SU is transmitting) and $A = \{1, 2, 3, 4, 5\}$ the rest of states. We develop a new convention and use the term "W" to refer to effective white spaces (i.e., the process is in B) and "NW" to non-white spaces (i.e., the process is in A). From hereon we use the idea of cycles. A cycle is the period that starts at the beginning of an NW period and continues with an uninterrupted NW period followed by an uninterrupted W period that ends just before another cycle starts, i.e. at the beginning of the next NW period.

Let X be the length of W we are looking for and Y the total length of time it takes to achieve $X \leq Y$. By this definition we are implying the X^{th} slot of W occurs at the Y^{th} time. Let J_1 be the phase at which the SU was at time 1 (when we started counting the W slots), and J_2 the phase at which it ended at time n (i.e., it finally obtained the desired m slots of white spaces).

First let us define two matrices

$$\begin{aligned} R(n) &= P_{A,A}^{n-1} P_{A,B}, \qquad \text{and} \\ S(m) &= P_{B,B}^{m-1} P_{B,A}, \qquad 1 \leq m \leq n. \end{aligned} \tag{25}$$

The element of $R(n)$ in row i and column j, $[R(n)]_{ij}$, is the probability that starting at time 1 at phase i of A, there is an uninterrupted NW period of duration n, which ends when the process enters B, through phase j, at time $n + 1$. Likewise, $[S(m)]_{ij}$, is the probability that starting at time 1 at phase i of B, there is an uninterrupted W period of duration m, which ends when the process enters A, through phase j, at time $m + 1$.

We define the matrix $C_\tau(m, n)$ that captures the probabilities that the τ^{th} cycle ($\tau \geq 2$) is n units long and the m^{th} W in this cycle occurs at the end of the cycle. It is clear that

$$C_\tau(m, n) = C_2(m, n) = R(n - m)S(m), \; m < n. \tag{26}$$

We now define a matrix $T_\tau^J(m, n)$ which represents the probability that the τ^{th} cycle ends at time n with a total of m W's so far (i.e., from the first cycle through the τ^{th} one), given that the first cycle started from states in $J = A, B$. It is straightforward to show that the following recursion applies:

$$T_{\tau+1}^J(m, n) = \sum_{m_1=1}^{m-1} \sum_{n_1=m_1+1}^{n-(m-m_1)} T_\tau^J(m-m_1, n-n_1) C_2(m_1, n_1), \qquad 1 \leq m \leq n, \quad \tau \leq \min(m, n-m+1) - 1, \quad J = A, B, \tag{27}$$

and it is initiated by

$$T_1^A(m,n) = \begin{cases} C_2(m,n), & m < n \\ 0, & m = n \end{cases}, \tag{28}$$

$$T_1^B(m,n) = \begin{cases} 0, & m < n \\ S(m), & m = n \end{cases}. \tag{29}$$

The organizational approach for implementing this recursion is as follows. Compute and store the matrices $C_2(v,w),\ v = 1,\cdots,m-1; w = v, v+1,\cdots,n-(m-1)$, and then compute the T matrices according to the recursion. For example, the first recursions are obtained as follows:

$$T_2^A(m,n) = \sum_{m_1=1}^{m-1} \sum_{n_1=m_1+1}^{n-(m-m_1)} T_1^A(m-m_1, n-n_1) C_2(m_1, n_1), \qquad 1 \le m \le n, \tag{30}$$

and

$$T_2^B(m,n) = \sum_{m_1=1}^{m-1} T_1^B(m-m_1, m-m_1) C_2(m_1, n-m+m_1), \qquad 1 \le m \le n. \tag{31}$$

After that the rest are obtained in the same manner using the general recursion equations.

We now turn our attention to the *last* cycle, that is, the cycle during which the total number of W we were looking for is attained, right at the end of the cycle or before it.

We define $\tilde{S}(m) = P_{B,B}^{m-1}$ and let

$$\tilde{C}_1(m,n) = R(n-m)\tilde{S}(m), \quad m < n. \tag{32}$$

The matrix $\tilde{C}_1(m,n)$ refers to the last cycle[2], and its interpretation is similar to that of $C_2(m,n)$ except that now the m^{th} W in the cycle can happen before the end of it. In a similar fashion, the interpretation of $\tilde{S}(m)$ is related to that of $S(m)$.

Let $p_{m,n}^{\tau}$ be the probability of obtaining exactly m slots of effective white spaces by the end of the n^{th} time slot in τ cycles. When $\tau = 1$, we have

$$p_{m,n}^1 = \begin{cases} \boldsymbol{\pi}_A R(n-m)\tilde{S}(m)\mathbf{1}, & m < n, \quad \text{(starting from the states in A)} \\ \boldsymbol{\pi}_B \tilde{S}(m)\mathbf{1}, & m = n, \quad \text{(starting from the states in B)} \end{cases}, \tag{33}$$

[2]The case in which there is exactly one cycle and hence the last cycle is also the first one is not considered here. This case is addressed separately in the expression of $p_{m,n}^1$ in (33)

and in general, for $\tau \geq 2$,

$$p_{m,n}^{\tau} = \begin{cases} 0, & n < m+\tau-1 \\ \sum_{m_1=1}^{m-1} \sum_{n_1=m_1+1}^{n-(m-m_1)} \boldsymbol{\pi}_B T_{\tau-1}^B(m-m_1, n-n_1)\tilde{C}_1(m_1,n_1)\mathbf{1}, & n = m+\tau-1 \\ \sum_{m_1=1}^{m-1} \sum_{n_1=m_1+1}^{n-(m-m_1)} \boldsymbol{\pi}_A T_{\tau-1}^A(m-m_1, n-n_1)\tilde{C}_1(m_1,n_1)\mathbf{1} & \\ \quad + \sum_{m_1=1}^{m-1} \sum_{n_1=m_1+1}^{n-(m-m_1)} \boldsymbol{\pi}_B T_{\tau-1}^B(m-m_1, n-n_1)\tilde{C}_1(m_1,n_1)\mathbf{1}, & n \geq m+\tau \end{cases}. \tag{34}$$

Let us denote by W_m the random variable that captures the waiting time in order to obtain a total of m slots of effective white space (obviously, $W_m \geq m$). Then, the probability of obtaining exactly m slots of effective white spaces by the end of the n^{th} time slot is given as

$$\mathrm{P}(W_m = n) = \mathrm{P}(X = m, Y = n) = \sum_{\tau=1}^{min(m,n-m+1)} p_{m,n}^{\tau}. \tag{35}$$

4.7. Waiting time for a minimum consecutive (uninterrupted) length of white spaces

Suppose a message requires a minimum of k uninterrupted time slots in order for it to be transmitted successfully. We want to determine the probability of having to wait t units of time in order for this event to occur.

First we define a matrix $D_\tau(m,n)$ as follows:

$$D_\tau(m,n) = \sum_{v=1}^{\min(m,n-1)} R(n-v)S(v), \qquad \tau \geq 2,\ m \geq 1,\ n \geq 2. \tag{36}$$

Here $D_\tau(m,n)$ captures the probability that the τ-th cycle (a general cycle other than the first one), which is n units long, has no more m white spaces. It is immediately clear that $D_\tau(m,n) = D_2(m,n),\ \tau \geq 2$.

We now define a matrix $F_\tau^J(m,n)$ which represents the probability that the τ^{th} cycle ends at time n without any of them having more than m consecutive white spaces, given it started from states in $J = A, B$. It is straightforward to show that the following recursions apply:

$$F_{\tau+1}^A(m,n) = \sum_{n_1=2}^{n-2\tau} F_\tau^A(m,n-n_1)D_2(m,n_1), \qquad \tau, m \geq 1,\ n \geq 2\tau,$$

$$F_{\tau+1}^B(m,n) = \sum_{n_1=2}^{n-(2\tau-1)} F_\tau^A(m,n-n_1)D_2(m,n_1), \qquad \tau, m \geq 1,\ n \geq 2\tau-1,$$

and they are initiated by

$$F_1^A(m,n) = D_2(m,n), \qquad n \geq 2,\ m \geq 1;$$

$$F_1^B(m,n) = S(n), \qquad n \geq m \geq 1.$$

Further define

$$H_0(m) = \sum_{v=m}^{\infty} S(v), \text{ and } H_j(m) = \sum_{v=m}^{\infty} R(j)S(v), \quad j \geq 1. \tag{37}$$

Hence, we have

$$H_0(m) = P_{B,B}^{m-1}(I - P_{B,B})^{-1}P_{B,A}, \quad \text{and} \tag{38}$$

$$H_j(m) = P_{A,A}^{j-1}P_{A,B}H_0(m). \tag{39}$$

Let us denote by W_m^c the random variable that captures the waiting time before receiving m consecutive slots of effective white space, that is, the m consecutive slots of white space come immediately after a waiting time of W_m^c slots. Then we have

$$\mathrm{P}(W_k^c = 0) = \boldsymbol{\pi}_B H_0(k)\mathbf{1}, \qquad k \geq 1$$

$$\mathrm{P}(W_k^c = t) = \boldsymbol{\pi}_A H_t(k)\mathbf{1} + \boldsymbol{\pi}_A \sum_{\ell=1}^{\lfloor \frac{t-1}{2} \rfloor} \sum_{v=0}^{t-2\ell} F_\ell^A(k-1, t-v)H_v(k)\mathbf{1} + \boldsymbol{\pi}_B \sum_{\ell=1}^{\lfloor \frac{t}{2} \rfloor} \sum_{v=0}^{t-(2\ell-1)} F_\ell^A(k-1, t-v)H_v(k)\mathbf{1}.$$

We observe that $H_0(m)\mathbf{1} = P_{B,B}^{m-1}\mathbf{1}$. Thus, we will be able to avoid computing the inverse matrix $(I - P_{B,B})^{-1}$.

5. Experimental Results and Discussion

Here we illustrate the versatility of the model developed in Section 3. To do that, we look at a number of aspects of the performance of the CR network, under different behaviors for both the PU and the SU.

Specifically, we examine: the efficiency of different sensing strategies in terms of throughput and energy consumption; the duration of uninterrupted effective white spaces; how does the distribution of the SU transmission length affect the interference caused to PUs, and the efficiency in using available white spaces; the waiting times until the SU transmits a given amount of data in total (through several transmissions separated by interruptions, or uninterruptedly).

Regarding the behavior of the SU, several distributions and a range of mean values have been considered for the durations of sensing and sleeping periods, and transmission. Our model also allowed us to study two different types of SU sensing capabilities: an SU which has two radios (and hence is able to sense the spectrum while it is transmitting), and an SU with a single radio that needs to stop transmitting to sense. With regard to the behavior after a transmission is interrupted by the detection of a PU, two possible alternatives have been contemplated: i) the SU can resume at the point at which it left off; ii) the transmission has to be started over from the beginning (non-resume transmission).

For the channel occupancy (PU behavior), we use the model proposed in [35], which is shown to exhibit a self-similar behavior over a finite but wide enough range of time-scales. Thus the block-matrices in (1) are given as,

$$D = \left[\begin{array}{c|ccccc} 1 - \sum_{k=1}^{n-1} \frac{1}{a^k} & \frac{1}{a} & \frac{1}{a^2} & \cdots & \frac{1}{a^{n-1}} \\ \hline \frac{b}{a} & 1 - \frac{b}{a} & & & \\ \left(\frac{b}{a}\right)^2 & & 1 - \left(\frac{b}{a}\right)^2 & & \\ \vdots & & & \ddots & \\ \left(\frac{b}{a}\right)^{n-1} & & & & 1 - \left(\frac{b}{a}\right)^{n-1} \end{array}\right], \tag{40}$$

Table 2: PU: basic scenario

	value
n	8
$\mathrm{E}[B]$	20
ρ_{PU}	0.5

Table 3: SU: basic scenario

	distribution	mean value
sleeping	geometric	10
sensing	deterministic	2
transmitting	geometric	10

	sensing errors	
	type 1	type 2
misdetection	$\phi_1 = 0.02$	$\phi_2 = 1$
false alarm	$\theta_1 = 0.08$	$\theta_2 = 0$

where n, a and b are the parameters of the model. The value of n determines the range of time-scales where the process can be considered as self-similar. Once the value of n is set, the values of a and b are obtained so as to fit the utilization factor of the channel $\rho_{\mathrm{PU}} = (1 - 1/b)/(1 - 1/b^n)$, and the average number of consecutive slots that the channel is busy (average burst length) $\mathrm{E}[B] = \left(\sum_{k=1}^{n-1} a^{-k}\right)^{-1}$.

Tables 2 and 3 show the settings of a *basic scenario*. Throughout all the numerical experiments presented here, all settings that are not explicitly mentioned are configured as in the basic scenario. Notice that in the basic scenario $\phi_2 = 1$, $\theta_2 = 0$, which means that sensing is not possible during transmission.

5.1. Impact of the sensing strategy

Different sensing strategies are evaluated in terms of the achieved throughput and energy consumption. Here, by sensing strategies we imply using different probability distributions and mean values for the duration of the *sleep* phase, that is, the duration of the interval between two consecutive sensing periods when the channel was sensed as busy in the first of them. We then assess which distribution performs best. Specifically, we consider the following distributions: deterministic, uniform, geometric and negative binomial (or Pascal). All these distributions —and basically any discrete distribution with a positive support— can be represented by discrete phase type distributions [22].

The impact of the sensing strategy on both η_t and η_s is shown in Fig. 1. As expected, by spending longer intervals in the sleeping state, η_t decreases and η_s increases. A similar representation is shown in Fig. 2. There we have plotted the results obtained from the numerical analysis of the model together with the results obtained through discrete-event computer simulations. As can be seen, analytical and simulation results match very closely. Other configurations not shown here yielded equally excellent agreement between analytical and simulation results. This validates the correctness of the analysis of the model, and in subsequent plots we will only represent the analytical results.

In Fig. 3 it can be observed that weighted efficiency (η) attains a maximum for short durations of the sleep period.

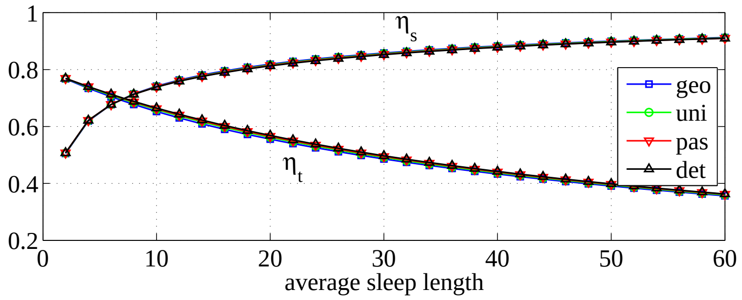

Figure 1: Efficiency of different sensing strategies. Lightly loaded channel, $\rho_{\mathrm{PU}} = 0.2$.

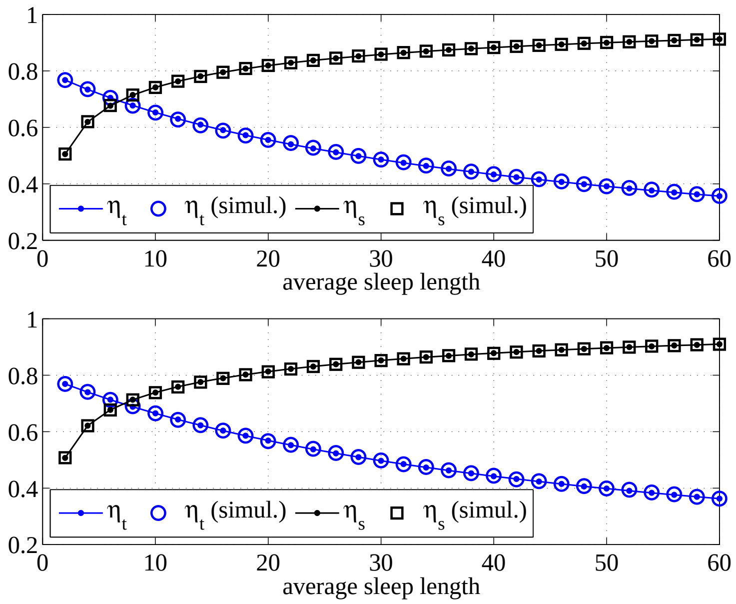

Figure 2: Efficiency of different sensing strategies: top, geometrically distributed sleep time; bottom, constant sleep time. Lightly loaded channel, $\rho_{\mathrm{PU}} = 0.2$; $n = 4$. Comparison of analytical and simulation results.

We also observe that as the channel gets more loaded the value of the efficiency decreases, and so does the value of the sleep duration at which the optimal efficiency is attained.

In the comparison of different distributions for the sleep duration (keeping constant the mean value) it was observed that lower variability results in better efficiency —in all our experiments the deterministic distribution yield the highest overall efficiency— although differences are very small.

The efficiency tradeoff is also explored in Fig. 4 where η_s and η_t are displayed one versus the other. There are three bundles of curves that correspond to different channel utilizations (high, $\rho_{\mathrm{PU}} = 0.8$; moderate, $\rho_{\mathrm{PU}} = 0.5$; low, $\rho_{\mathrm{PU}} = 0.2$). The four curves in each bundle correspond to the four distributions of the sleep time, and along each curve the mean sleep time is varied. For a given value of the mean sleep time, the value of η_s is approximately the same independently of the channel load. However, for a fixed η_s, η_t decreases substantially as the load increases.

The impact of the sensing strategy on the distribution of the duration of an effective white space is shown in

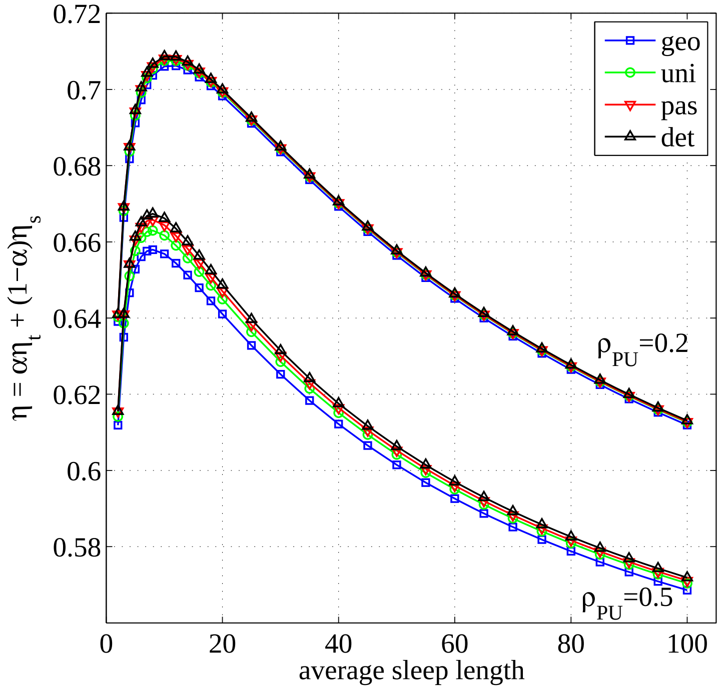

Figure 3: Overall efficiency of different sensing strategies; $\alpha = 0.5$. Lightly loaded channel, $\rho_{\rm PU} = 0.2$; moderately loaded channel, $\rho_{\rm PU} = 0.5$.

Fig. 5. Other experiments not shown here show that differences across distributions were not noticeable for this type of representation. Thus, in Fig. 5 we only represented the results for the deterministic distribution of sleep length. For each channel load two different average sleep durations are represented: the one at which the weighted efficiency peaks in Fig. 3 and another one four times larger.

5.2. Interference caused to PUs

Figure 6 shows the impact of the transmission length on the interference caused to PUs (I_f). The efficiency factor η_t is also plotted to illustrate the existing tradeoff between interference and efficiency. The comparison of different distributions of the transmission length reveals that less variability yields lower interference (I_f), whereas has little or no impact on η_t. As expected, the longer the duration of the transmission the higher I_f and also η_t, but the impact on the latter is less noticeable. It is also observed that increasing the duration of the sensing phase reduces the amount of interference, but reduces the efficiency as well.

The effect of having an extra radio so that the SU can sense and transmit simultaneously ($\phi_2 = 0.02$, $\theta_2 = 0.08$) is illustrated in Fig. 7. As observed, sensing during transmission reduces the interference caused to PUs dramatically. Increasing the duration of the sensing phase has a positive effect in the amount of interference, and this effect is as pronounced as in the case where sensing was not carried out during transmission. Compared with Fig. 6 the curves for I_f are relatively flat, in other words, the duration of the transmission has a lower impact on the interference when the SU can transmit and sense simultaneously. It is interesting to see that when the transmission length is high the value

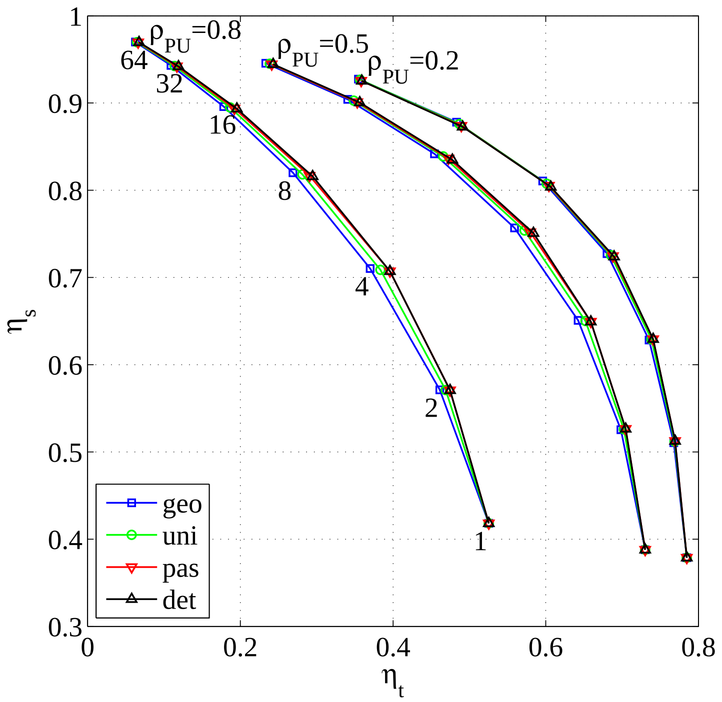

Figure 4: Tradeoff between η_t and η_s. The numbers next to the points denote the average sleep length for the nearby set of points.

of η_t is slight lower when sensing is done during transmission (compare the curves in Figs. 6 and 7). This somewhat counterintuitive observation can be attributed to the fact that when the SU can sense during transmission that sensing is affected by false alarm errors, which can make the SU to abort transmissions and miss opportunities to use an idle channel.

5.3. Probabilities of global misdetection and global false alarm

Here we study how the duration of the sensing period affects the probabilities of global misdetection (P_{md}) and false alarm (P_{fa}). In Fig. 8 we plotted both probabilities against the number of slots in a sensing period for different channel loads.

As expected, the more slots there are in a sensing period, the lower P_{md} is and the higher P_{fa} is. However, P_{md} decreases more rapidly than P_{fa} increases, which is a positive characteristic. Furthermore, the increasing trend of the false alarm probability slows down and its value tends to stabilize as the sensing length increases, whereas the misdetection probability declines steadily. It can also be observed a well-defined knee in the curves for P_{md} located at low values of the sensing length (2-3 time slots).

5.4. Efficiency when incomplete transmissions are restarted

We now study the transmission efficiency when transmissions that are not completed successfully have to be restarted from the beginning. Two graphs corresponding to different values of ϕ_1 and θ_1 are shown in Fig. 9. In each graph we represent the transmission efficiency (η_g, defined in (24)) and the interference factor (I_f, defined in (14)) as

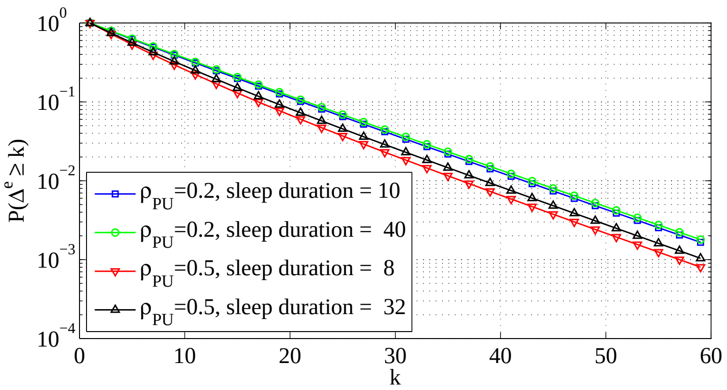

Figure 5: Complementary CDF of the duration of an effective white space. Deterministic sleep duration

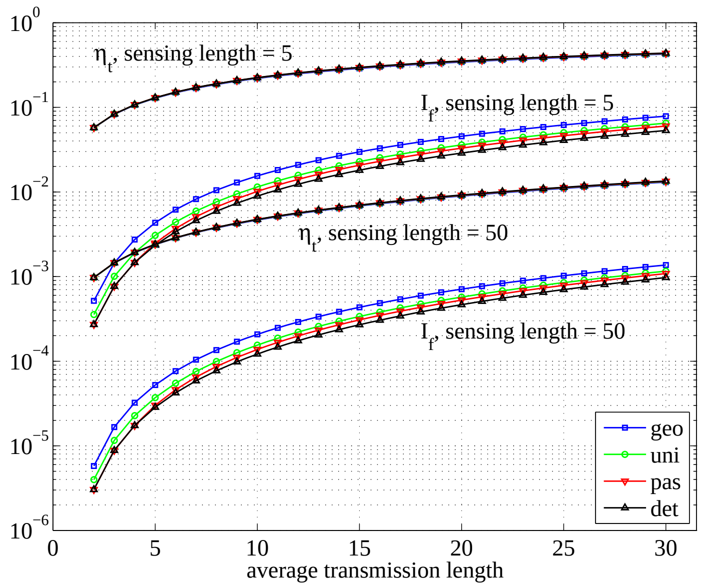

Figure 6: Caused interference and transmission efficiency. No sensing during transmission.

functions of the transmission length (i.e, the message length); a deterministic distribution of the transmission length is used in this case. Besides, in each graph we plot results for the case in which the SU cannot sense during transmission ($\phi_2 = 1$, $\theta_2 = 0$), and also for the case where sensing during transmission is possible and has the same accuracy as when the SU is not transmitting ($\phi_2 = \phi_1$, $\theta_2 = \theta_1$).

The qualitative behavior of I_f is roughly the same in both graphs. When the SU senses during transmission the interference factor experiences a negligible increase as the transmission length grows and it is maintained a very low value. When the SU cannot sense during transmission the interference factor increases linearly with the transmission length.

In both plots we see that the transmission efficiency is lower when the SU can sense during transmission. This occurs because when sensing is done during transmission false alarms may occur. If this happens, the SU stops

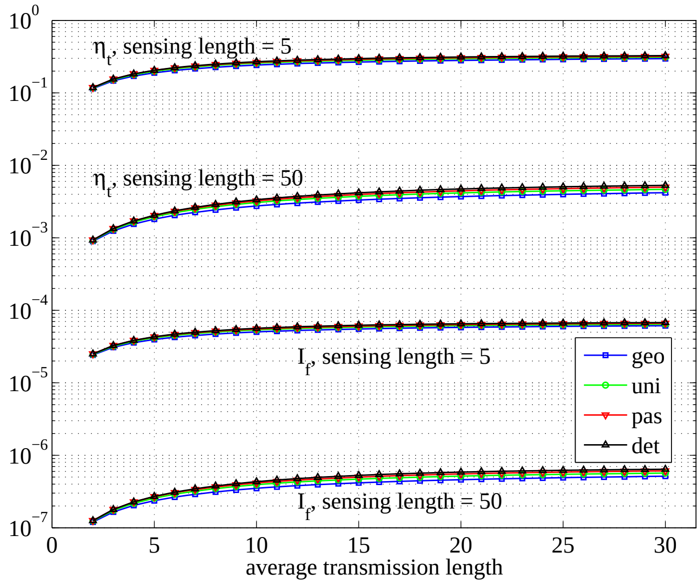

Figure 7: Caused interference and transmission efficiency. SU can sense and transmit simultaneously.

(unnecessarily) the ongoing transmission and all data transmitted up to then in the current transmission is rendered useless. Moreover, the gap between the two curves widens as the message length increases, since the probability of the whole message being transmitted successfully at once decreases with the length of the message. It is also observed that, in the four curves, as the message length grows η_g rises rapidly, reaches a peak and then declines. This shapes can be explained as the result of two competing effects on efficiency. While lengthier messages are more efficient owing to the fixed cost of the sensing period before starting to transmit, on the other hand lengthier messages make it less likely that the message transmission can be completed before the PU shows up. Additionally, as already mentioned, when sensing is carried out during transmission a lengthier message implies a higher probability of aborting its transmission because of a false alarm. This explains that, for the case in which sensing is carried out during transmission (*configuration 2*), the decline of η_g starts earlier and is steeper on the left plot (in which the value of $\theta_2 = \theta_1$ is higher).

As it has already been noted, sensing during transmission permits to keep interference caused to the PU at low values independently of the transmission length, which in turn would allow longer SU transmissions. However, the above observations seem to suggest that if interrupted transmissions have to be restarted from the beginning the advantage of being able to sense during transmission may vanish due to the effect of false alarms.

5.5. *Waiting times*

Figures 10 and 11 display, respectively, the curves for $\log \mathrm{P}(W_{20} > k)$ and $\log \mathrm{P}(W_5^c > k)$, with different loads ρ_{PU} and time-span (n) in the correlations of the channel characteristics. The mean duration of sleeping and transmission were both set to 5 slots.

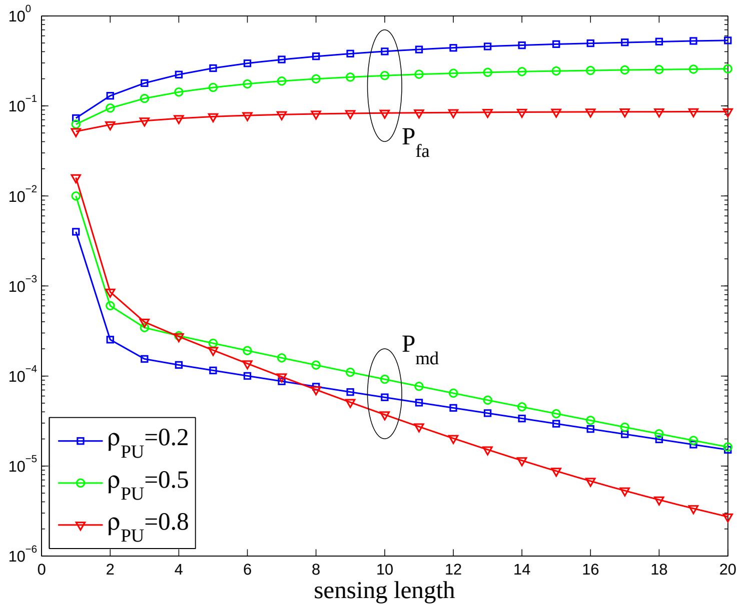

Figure 8: Impact of the duration of the sensing period on the probabilities of global misdetection and global false alarm.

We observe a rather steep initial decrease and then a steady decline, which is approximately linear in logarithmic representation. The depth and the duration of the initial decrease depends on the load of the channel: the least loaded the channel the longer and sharper the decrease is. The span of the time correlations in the channel dynamics (given by the parameter n) has no impact on the initial decrease. On the other hand, the slope of the linear decay is affected by both the channel load and the value of n.

6. Conclusion

In this paper we have developed mathematical models for the study of sensing strategies in Cognitive Radio networks. The models developed here are largely general. The channel busy and idle periods are described as alternating Markov phase renewal process, which allows the busy and idle times durations to assume a wide variety of distributions and also captures broad correlation aspects of the two intervals. As regards the secondary user (SU) behavior, the duration of transmissions, sensing periods and the intervals between consecutive sensing periods (sleeping) are modeled by general *phase type* distributions. Furthermore, imperfect sensing has been considered by modeling two types of misdetections and false alarms, which also allowed us to cover a wide range of situations.

Several key performance measures in Cognitive Radio network have been obtained from the analysis of our model. Most notably, we derived the distribution of the length of an *effective white space*, the distributions of the waiting times until the SU transmits a given amount of data in total (through several transmissions separated by interruptions, or uninterruptedly), and the SU *goodput* when an interrupted SU transmission has to be started over from the beginning. We also obtained expressions for the probabilities of *global misdetection* and *global false alarm*, which have a more

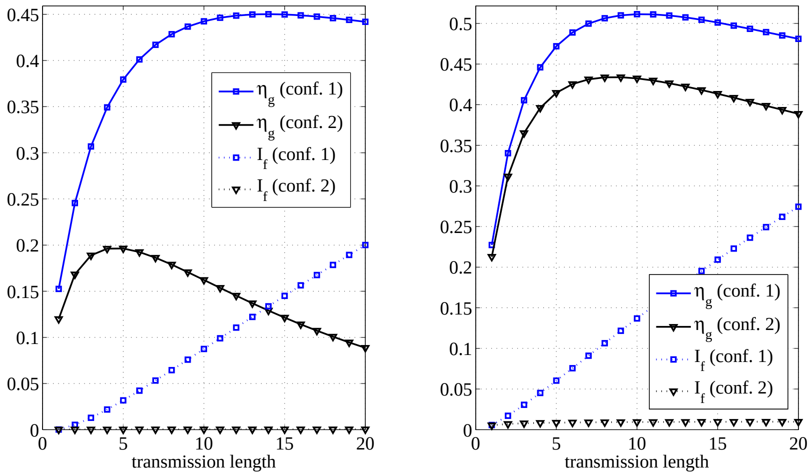

Figure 9: Efficiency and interference with non-resume transmission. Configuration 1: no sensing during transmission ($\phi_2 = 1$, $\theta_2 = 0$). Configuration 2: sensing during transmission ($\phi_2 = \phi_1$, $\theta_2 = \theta_1$). Left: $\phi_1 = 0.02$, $\theta_1 = 0.08$. Right: $\phi_1 = 0.20$, $\theta_1 = 0.01$.

general meaning that the probabilities of misdetection and false alarm referred to measurements taken during a single time slot.

Acknowledgements

The research of A. S. Alfa was partially supported by the NSERC (Natural Sciences and Engineering Research Council) of Canada under Grant G00315156. Most of the contribution of V. Pla was done while visiting the University of Manitoba. This visit was supported by the *Ministerio de Educación* of Spain under grant PR2011-0055, and by the UPV through the *Programa de Apoyo a la Investigación y Desarrollo (PAID-00-12)*. The research of the authors from the Universitat Politècnica de València was partially supported by the Ministry of Economy and Competitiveness of Spain under Grant TIN2013-47272-C2-1-R.

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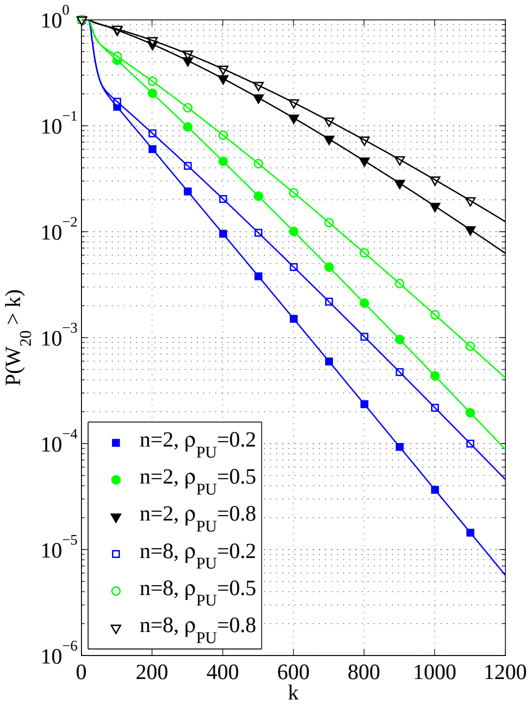

Figure 10: Complementary CDF of the waiting time for a total of 20 slots of white space.

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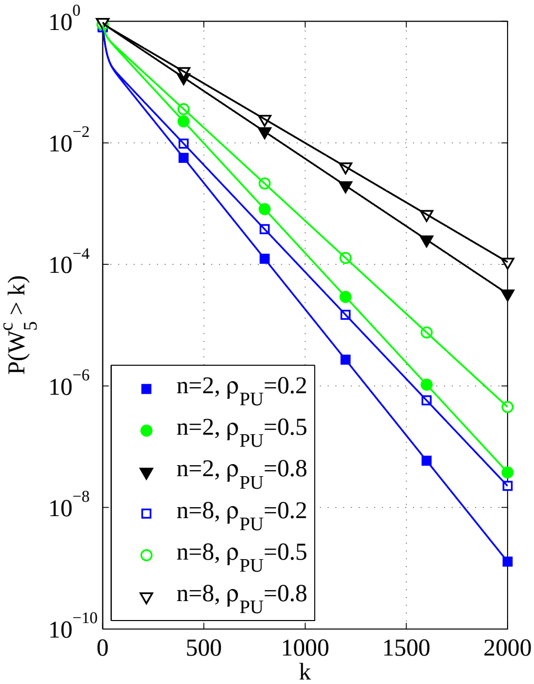

Figure 11: Complementary CDF of waiting time before getting 5 consecutive slots of white space.

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Appendix A. Detailed transition matrix

$$P = \begin{bmatrix} D_b \otimes L & D_b \otimes (\boldsymbol{l\beta}) & 0 & d_{bi} \otimes L & d_{bi} \otimes (\boldsymbol{l\beta}) & 0 \\ \bar{\phi}_1 D_b \otimes (\mathbf{1}\boldsymbol{\delta}) & \phi_1 D_b \otimes S & \phi_1 D_b \otimes (\boldsymbol{s\alpha}) & \bar{\phi}_1 d_{bi} \otimes (\mathbf{1}\boldsymbol{\delta}) & \phi_1 d_{bi} \otimes S & \phi_1 d_{bi} \otimes (\boldsymbol{s\alpha}) \\ \bar{\phi}_2 D_b \otimes (\mathbf{1}\boldsymbol{\delta}) & \phi_2 D_b \otimes (\boldsymbol{t\beta}) & \phi_2 D_b \otimes T & \bar{\phi}_2 d_{bi} \otimes (\mathbf{1}\boldsymbol{\delta}) & \phi_2 d_{bi} \otimes (\boldsymbol{t\beta}) & \phi_2 d_{bi} \otimes T \\ d_{ib} \otimes L & d_{ib} \otimes (\boldsymbol{l\beta}) & 0 & D_i \otimes L & D_i \otimes (\boldsymbol{l\beta}) & 0 \\ \bar{\phi}_1 d_{ib} \otimes (\mathbf{1}\boldsymbol{\delta}) & \phi_1 d_{ib} \otimes S & \phi_1 d_{ib} \otimes (\mathbf{s}\boldsymbol{\alpha}) & \theta_1 D_i \otimes (\mathbf{1}\boldsymbol{\delta}) & \bar{\theta}_1 D_i \otimes S & \bar{\theta}_1 D_i \otimes (\boldsymbol{s\alpha}) \\ \bar{\phi}_2 d_{ib} \otimes (\mathbf{1}\boldsymbol{\delta}) & \phi_2 d_{ib} \otimes (\boldsymbol{t\beta}) & \phi_2 d_{ib} \otimes T & \theta_2 D_i \otimes (\mathbf{1}\boldsymbol{\delta}) & \bar{\theta}_2 D_i \otimes (\boldsymbol{t\beta}) & \bar{\theta}_2 D_i \otimes T \end{bmatrix}, \tag{A.1}$$

Matrix (A.1) provides the detailed expressions for the blocks in the transition matrix P introduced in (5). For illustration purposes, we explain some of them below. For the rest, a similar reasoning would apply. We deal with probabilities in *matrix form*. Here again, we refer to the matrix blocks as probability matrices of going from one state (which is composed of several phases) to another state (which is also composed of several phases), so that the row and the column of the block matrix represent the phase in the initial and final state, respectively.

The detailed expressions for the blocks of the transition matrix P are shown in Eq. (A.1), where $\boldsymbol{l}, \boldsymbol{s}$ and $\boldsymbol{t}$ are, respectively, the absorption probability vectors associated with sleeping, sensing and transmitting.

- $P_{11} = D_b \otimes L$: the channel keeps busy and the SU continues sleeping.
- $P_{12} = D_b \otimes (\boldsymbol{l\beta})$: the channel keeps busy, and the SU finishes sleeping and starts sensing.
- $P_{13} = P_{16} = P_{43} = P_{46} = 0$ since the SU cannot go directly from sleeping to transmit; it must carry out a sensing period first.
- $P_{14} = d_{bi} \otimes L$: the channel switches from busy to idle and the SU continues sleeping.
- $P_{15} = d_{bi} \otimes (\boldsymbol{l\beta})$: the channel switches from busy to idle, and the SU finishes sleeping and starts sensing.
- $P_{21} = \bar{\phi}_1 D_b \otimes ((S\mathbf{1} + \boldsymbol{s})\boldsymbol{\delta}) = \bar{\phi}_1 D_b \otimes (\mathbf{1}\boldsymbol{\delta})$: the channel keeps busy and it is correctly detected, then the SU starts a sleeping cycle no matter whether the sensing period had terminated or the phase at which it was.
- $P_{22} = \phi_1 D_b \otimes S$: the channel keeps busy but a misdetection occurs and the SU continues the sensing period.
- $P_{23} = \phi_1 D_b \otimes (\boldsymbol{s\alpha})$: the channel keeps busy but a misdetection occurs and the SU, which had finished with the sensing period, starts transmitting.
- $P_{24} = \bar{\phi}_1 d_{bi} \otimes ((S\mathbf{1} + \boldsymbol{s})\boldsymbol{\delta}) = \bar{\phi}_1 d_{bi} \otimes (\mathbf{1}\boldsymbol{\delta})$: the channel was initially busy and it is correctly detected, then the SU starts a sleeping cycle no matter whether the transmission had terminated or the phase at which it was.
- $P_{25} = \phi_1 d_{bi} \otimes S$: although the channel was initially busy a misdetection occurs and the SU continues the sensing period.

- $P_{26} = \phi_1 d_{bi} \otimes (\boldsymbol{s\alpha})$: although the channel switches to busy a misdetection occurs and the SU, which had finished with the sensing period, starts transmitting.
- $P_{31} = \bar{\phi}_2 D_b \otimes ((T\mathbf{1} + \boldsymbol{t})\boldsymbol{\delta}) = \bar{\phi}_2 D_b \otimes (\mathbf{1}\boldsymbol{\delta})$: the channel keeps busy but the SU, which was transmitting because of a previous misdetection, detects it and starts a sleeping cycle.
- $P_{32} = \phi_2 D_b \otimes (\boldsymbol{t\beta})$: the channel keeps busy and the SU, which was transmitting because of a previous misdetection, still fails to detect it, finish its transmission and starts a sensing period.
- $P_{33} = \phi_2 D_b \otimes T$: the channel keeps busy and the SU, which was transmitting because of a previous misdetection, still fails to detect it and keeps transmitting.
- $P_{34} = \bar{\phi}_2 d_{bi} \otimes ((T\mathbf{1} + \boldsymbol{t})\boldsymbol{\delta}) = \bar{\phi}_2 d_{bi} \otimes (\mathbf{1}\boldsymbol{\delta})$: the channel was initially busy and the SU, which was transmitting because of a previous misdetection, happens to detect it now correctly; then the SU starts a sleeping cycle no matter whether the transmission had terminated or the phase at which it was.
- $P_{35} = \phi_2 d_{bi} \otimes (\boldsymbol{t\beta})$: the channel was initially busy and the SU, which was transmitting because of a previous misdetection, still fails to detect it, finishes its transmission and starts a sensing period.
- $P_{36} = \phi_2 d_{bi} \otimes T$: the channel was initially busy and the SU, which was transmitting because of a previous misdetection, still fails to detect it and keeps transmitting.