Economía Agraria y Recursos Naturales. ISSN: 1578-0732. e-ISSN: 2174-7350. Vol. 15,2. (2015). pp. 5-30

Analysing the economic value of meteorological information to improve crop risk management decisions in a dynamic context

Emilio Cerdá[a] and Sonia Quiroga[b]

ABSTRACT: We evaluate the added value of a forecast service that can provide probabilistic predictions for adverse weather events for two differentiated seasons, corresponding to the same productive cycle. The paper builds on a cost-loss dynamic model, by considering the role of forecasting systems in the decision making process. We present the analytical solution for this problem which is consistent with the numerical results in the literature. However, we prove that there is a range of regions for the optimal policy depending on the cost of crop protection, the avoided loss and the quality of the information available. Finally, we illustrate the results with a numerical example.

KEYWORDS: Cost-loss ratio, Crop yield protection, dynamic decision models, climate risks.

JEL classification: C61, Q12.

DOI: 10.7201/earn.2015.02.01.

Análisis del valor económico de la información meteorológica utilizada en la toma de decisiones sobre cultivos, en un contexto dinámico de gestión de riesgos

RESUMEN: Evaluamos el valor añadido de los servicios meteorológicos que proporcionan predicciones probabilísticas para eventos climáticos adversos considerando dos estaciones diferentes, correspondientes a un mismo ciclo productivo. Este artículo se apoya en un modelo coste-pérdida dinámico para considerar el papel de los sistemas de predicción y alerta temprana en los procesos de toma de decisión. Se presenta la solución analítica para este problema, que es consistente con los resultados numéricos en la literatura. Sin embargo, probamos que la política óptima presenta numerosas regiones distintas dependiendo del coste de proteger, la pérdida evitada y la calidad de la información disponible. Finalmente, ilustramos los resultados con un ejemplo numérico.

PALABRAS CLAVE: Protección del rendimiento del cultivo, ratio coste-pérdida, modelos dinámicos de decisión, riesgos climáticos.

Clasificación JEL: C61, Q12.

DOI: 10.7201/earn.2015.02.01.

---

[a] Dpto. Fundamentos del Análisis Económico I. Universidad Complutense de Madrid.

[b] Dpto. Economía. Universidad de Alcalá.

Acknowledgements: We thank Barry Readman for the linguistic revision of the text. Emilio Cerdá thanks funding from the Spanish Ministry of Economía and Competitividad through the research project ECO2012-39553-C04-01.

Corresponding author: *Emilio Cerdá. E-mail: ecerdate@ccee.ucm.es*.

Received on June 2013. Accepted on February 2015.

1. Introduction

Great concern exists about the increase in adverse climate events and their consequences for society and the economy (IPCC, 2007). Agriculture is one of the most affected economic sectors suffering each year high losses due to meteorological changes (Ciscar *et al.*, 2011; Battisti and Naylor, 2009; Olesen and Bindi, 2002) and water conflicts, especially in the Mediterranean region (Gómez-Limón and Riesgo, 2004; Iglesias *et al.*, 2009). Nevertheless, improved farming management can largely cope with the climate risks and so crop production is one of the activities with the highest adaptive capacity to climate change effects (EU COM, 2009).

Adaptation policies on a global scale are essential to cope with the potential losses, but the development of private adaptation strategies based on improved management is also crucial (Brouwer *et al.*, 2004). For this kind of adaptation measures, meteorological information can play an important role if it assists the improvement of management decisions. Nowadays, a number of high quality seasonal forecasts and weather forecasts are available leading to significant applications in agricultural management. This is more so since a correlation between ENSO (El Niño/Southern Oscillation) and rainfall and temperature patterns is being widely analysed.

In this paper we analyse the economic value of information on adverse climate events that some forecasting service is giving in advance with differentiated probabilistic predictions. We consider two time periods, corresponding to the same productive cycle, and evaluate the optimal decision making about protective actions to avoid potential losses. Then we analyse how the information improves the decision making process.

Dynamic treatment is essential for this kind of risk management problems. When farmers decide about protecting harvests from a meteorological risk, they do not obtain immediate results, since the decision taken over every period influences the total results. To take no protective action in one period is enough to risk losing the overall harvest if the adverse weather event occurs. For example, frost protection is just one phase of an orchard management process, but if the crop is lost due to frost, the decisions pertinent to orchard operation for the rest of the time horizon under consideration may have limited or no effect on current year production (Cerdá and Quiroga, 2011).

The cost-loss model has been widely used to analyse this kind of management problem (Katz and Murphy, 1997; Cerdá and Quiroga, 2010; Wilks, 1997; Palmer, 2002), and several examples with numerical results have been presented for a farmer's protection in a dynamic framework (Katz, 1993; Meza *et al.*, 2003; Katz and Ehrendorfer, 2006). Analytical results for the optimal policy of the dynamic model for the case of climatological information have been calculated in Cerdá and Quiroga (2011). This paper builds on this dynamic model considering the role of forecasting systems in the decision making process.

The structure of the decision making problem is as follows: (i) the farmer can decide to protect or not to protect in each of the two periods, (ii) adverse meteorological events can occur in either of the two periods considered (corresponding to the same productive cycle) and they can cause crop losses if the protective action has not been taken, (iii) the farmer makes the decision about protection in each of the two periods before knowing if severe weather is coming, (iv) the information available is of two types. One kind of information is the climatological information, based on historical records of climate variables (ie. the statistical probability of drought for the considered region). We consider that this is always available and known for the farmer´s community. Another type of information is the weather forecast or seasonal forecast information. This forecast system is considered as a kind of early warning system that allows the farmer to have improved information about the probability of severe events.

This paper is structured as follows: the next section presents the model structure and equations. Analytical solutions are obtained in Section 3, while in Section 4 an illustrative numerical example is developed. Finally some conclusions are presented in Section 5.

2. The model

The model involves two possible actions, to protect ($\alpha = 1$), or not to protect ($\alpha = 0$), and two possible events, adverse weather ($\theta = 1$) and no adverse weather ($\theta = 0$). If protective action is taken, the farmer is assumed to incur a cost, $C > 0$ that we consider a proportion of the avoided loss, $C = \gamma L$ where $0 < \gamma < 1$. A loss $L > 0$ is suffered by the farmer if protective action is not taken and severe adverse weather occurs. There is no cost or loss, otherwise. The type of protection could avoid the physical loss (i.e. by applying protective technology to some plants) or just provide an economic compensation (i.e. purchasing insurance for part of the crop). We consider the common assumptions of the familiar prototype problem usually referred to as the cost-loss ratio situation (Katz and Murphy, 1997; Katz, 1993) and a summary of the model variables and parameters is presented in Table 1.

TABLE 1

Description of the variables and parameters included in the model

Name	Variable or parameter
θ_i	State of weather in the i-period ($\theta = 1$, “adverse weather”; $\theta = 0$, “no adverse weather”)
α_i	Protection level for the harvest (0, no protection or 1, total protection) in the period i
L_i	Amount of harvest available at the beginning of the period i
γ	Cost of the protection measures (proportion of the avoided loss: $C_i = \gamma L_i$)
P_θ	Climatological information: Pr [$\theta = 1$]

Source: Own work.

When only the climatological information is available, the optimization problem to minimize the expected expenses deriving from the adverse weather event (i.e. severe drought) in the two periods can be written as follows:

$$\min_{\{\alpha_1,\alpha_2\}} E\{\gamma\alpha_1 L_1 + (1-\alpha_1)\theta_1 L_1 + \gamma\alpha_2 L_2 + (1-\alpha_2)\theta_2 L_2\}$$

$$\begin{aligned} S.t. \quad & L_3 = L_2 - \theta_2(1-\alpha_2)L_2 \\ & L_2 = L_1 - \theta_1(1-\alpha_1)L_1 \\ & L_1 = L \\ & \alpha_1 \epsilon \{0,1\}, \qquad for\ i = 1,2, \end{aligned} \qquad [1]$$

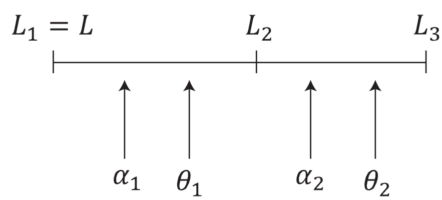
θ_1 and θ_2 are independent random variables and take the value 1 with probability P_θ and the value 0 with probability 1-P_θ.

L_1 is predetermined as L, $L_2 \in \{L,0\}$ and $L_3 \in \{L,0\}$.

This is a stochastic, dynamic optimization problem of two time periods in discrete time, which will be solved using the method of Dynamic Programming. In this problem, $L_i = (i = 1, 2, 3)$ is the state variable and α_i $(i = 1, 2)$ is the control variable. The sequence in which the state variable, the control variable and also the stochastic component of the problem θ_i $(i = 1, 2)$ appear, is represented in Figure 1. This sequence is very helpful for the understanding of the problem and also for the calculation of the optimal solution, using the Bellman equations.

FIGURE 1

Sequence in which the different variables appear in Problem [1]



Source: Own work.

In the case of counting on an information system for weather forecasting or seasonal forecasting, the farmer's decision making can be potentially improved, so we want to analyze the optimal policy also in this case. As in Murphy *et al*. (1985), we consider the incorporation of additional information to the model which is introduced as an imperfect weather or seasonal forecasting from a meteorological office. It is assumed that the farmer receives an imperfect forecast about the state of nature at the beginning of each of the two periods. For $i \in \{1,2\}$, this information is represented by the variable Z_i which indicates a forecast of adverse weather ($Z_i = 1$), or of non-adverse weather ($Z_i = 0$). The conditional probabilities of adverse weather are denoted by $P_1 = \Pr\{\theta_i = 1 / Z_i = 1\}$ and $P_0 = \Pr\{\theta_i = 1 / Z_i = 1\}$. In addition, as in Murphy *et al.* (1985) it is assumed that Pr $\{Z_i = 1\}$= Pr $\{\theta_i = 1\}$=P_θ, that is, the forecasting system produces adverse weather signals with the same probability that adverse weather events take place. As Katz and Murhy (1997: 187-188) explain, concerning this assumption, "*It is assumed that forecasts of adverse weather are issued with the same long-run relative frequency as the occurrence of adverse weather (termed "overall reliability" or "unconditionally unbiased"). This requirement is equivalent to constraining the conditional probability P_0 to move from P_θ toward zero at the same relative rate at which the conditional probability P_1 moves from P_θ toward one*".

Without loss of generality, $0 \le P_0 \le P_\theta \le P_1 \le$, is also assumed. In these conditions it is easily obtained[1] that $P_0 = \frac{(1-P_1)P_\theta}{(1-P_\theta)}$.

The decision making problem for the two periods can be expressed as the following stochastic, dynamic optimization problem:

$$\begin{aligned}
&\underset{\{\alpha_1,\alpha_2\}}{Min} \quad \underset{Z_1,\theta_1,Z_2,\theta_2}{E}\left\{\sum_{i=1}^{2}\left[\gamma\alpha_i L_i + (1-\alpha_i)\theta_i L_i\right]\right\} \\
&s.t. \quad L_3 = L_2 - \theta_2(1-\alpha_2)L_2 \\
&\qquad L_2 = L_1 - \theta_1(1-\alpha_1)L_1 \\
&\qquad L_1 = L \\
&\qquad \alpha_i \in \{0,1\}, \qquad for \quad i = 1,2,
\end{aligned} \qquad [2]$$

where Z_i is a random variable that can take the values 1 or 0 with the following associated probabilities: $\begin{cases} 1 & \text{with prob } P_\theta \\ 0 & \text{with prob } 1-P_\theta \end{cases}$.

θ_i is also a random variable that can take the values 1 or 0 with the following associated probabilities depending on the forecast value:

$$\begin{cases} \text{If} \quad Z_i = 1, \quad \theta_i = \begin{cases} 1 & \text{with prob } P_1 \\ 0 & \text{with prob } 1-P_1 \end{cases} \\ \text{If} \quad Z_i = 0, \quad \theta_i = \begin{cases} 1 & \text{with prob } P_0 \\ 0 & \text{with prob } 1-P_0 \end{cases} \end{cases}$$

As before, we assume that θ_1 y θ_2 are independent random variables.

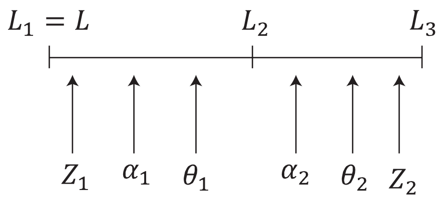
---

[1] $P_\theta = \Pr\{\theta_i = 1\} = \Pr\{\theta_i = 1 / Z_i = 1\}\Pr\{Z_i = 1\} + \Pr\{\theta_i = 1 / Z_i = 0\}\Pr\{Z_i = 0\} = P_1 P_\theta + P_0(1-P_\theta) \Rightarrow P_0 = \frac{(1-P_1)P_\theta}{1-P_\theta}$.

The sequence of the variables in the decision making problem [2] is shown in Figure 2.

FIGURE 2

Sequence in which the different variables appear in Problem [2]



Source: Own work.

3. Results

In this section the two cases, climatological information and additional information consisting of imperfect weather or seasonal forecasting from a meteorological office, are studied independently.

3.1. Optimal policy when only climatological information is available

Proposition 1 shows the optimal policy for the farmer when only climatological information is available, that is when the farmer knows about the historical probability but does not possess a forecast system.

Proposition 1

In the optimization problem [1], corresponding to the case of climatological information[2]:

i) If $\gamma < \frac{P_\theta}{P_\theta + 1}$, the optimal decision is $\alpha_1 = 1$ and $\alpha_2 = 1$ that is to protect in both periods. In this case, the minimum expense is: $J_1^* \{L\} = 2\gamma L$.

ii) If $\frac{P_\theta}{P_\theta + 1} \leq \gamma < P_\theta$, the optimal decision is $\alpha_1 = 1$ and $\alpha_2 = 1$. That is to protect in the second period and not to protect in the first period. The optimal expense in this case is: $J_1^* \{L\} = P_\theta L + \gamma(1 - P_\theta)L$.

[2] It is interesting to observe that it will never be optimal to protect in the first period and not to protect in the second period $\alpha_1 = 1$ and $\alpha_2 = 1$. Once the farmer incurs the protective cost, he needs to be sure of not suffering the loss in the second period.

iii) If $\gamma \geq P_0$, the optimal decision is $\alpha_1 = 1$ and $\alpha_2 = 1$, so it is optimal not to protect in each of the periods. The optimal level of expense in this case is: $J_1^*\{L\} = P_0 L + (1-P_0)P_0 L$.

The proof is in the Annex.

Comparison with the static case:

In the static case (one period), the optimal policy when only climatological information is available is the following: to protect if $\gamma < P_0$, and not to protect if $\gamma \geq P_0$. That is, if the cost of protection is high it is better not to protect and if that cost is low it is better to protect, P_0 being the threshold which separates both possibilities.

The optimal policy obtained in the two-period case is less protective than that corresponding to the static case. When the condition for no protection in the static case holds ($\gamma \geq P_0$), it is optimal not to protect in each of the two periods in the dynamic case. However the protective situation in the static model ($\gamma < P_0$) is decomposed in two cases in the dynamic model: to protect in each of the two periods if the cost of protection is very low $\left(\gamma < \frac{P_0}{1+P_0}\right)$ and not to protect in the first period but protect in the second one if the cost of protection takes some intermediate value $\left(\frac{P_0}{1+P_0} \leq \gamma < P_0\right)$.

3.2. Optimal policy when meteorological information is available

In this subsection the situation in which additional meteorological information is introduced is analysed. The optimal policy and the corresponding minimum expense for each possibility are obtained and stated in Proposition 2.

Different mathematical expressions for the optimal policy appear depending on $P_0 \leq \frac{P_1}{1+P_1}$ or $P_0 > \frac{P_1}{1+P_1}$. Therefore the optimal policy is obtained for each of the two cases. As $P_0 \leq P_1$, it is possible that $P_0 \leq \frac{P_1}{1+P_1}$ (for example for $P_0 = 0.4$ and $P_1 = 0.75$) and it is also possible that $P_0 > \frac{P_1}{1+P_1}$ (for example for $P_0 = 0.4$ and $P_1 = 0.6$).

As $\frac{d}{dP_1}\left(\frac{P_1}{1+P_1}\right) = \frac{1}{(1+P_1)^2} > 0$, the higher the value of P_1, the higher the value of $\frac{P_1}{1+P_1}$. On the other hand, as $P_0 = \frac{(1-P_1)P_0}{(1-P_0)}$, the higher the value of P_1, the lower the value of P_0. Therefore, if P_1 is high (which means that the weather forecast or the seasonal forecast system is good), the condition $P_0 \leq \frac{P_1}{1+P_1}$ will hold.

Proposition 2

(A) Let us assume that $P_0 \le \dfrac{P_1}{1+P_1}$ *(high quality of the meteorological information). The optimal policy for the minimization problem in [2] is*[3]*:*

i) If $\gamma < \dfrac{P_0}{1+P_0}$, $\begin{cases} \alpha_1^* = 1 \\ \alpha_2^* = 1 \end{cases}$

and $J_1^*\{L\} = 2\gamma L$.

ii) If $\dfrac{P_0}{1+P_0} \le \gamma < P_0$, $\begin{cases} \alpha_1^* = 1, & \text{if } Z_1 = 1 \\ \alpha_1^* = 0, & \text{if } Z_1 = 0 \\ \alpha_2^* = 1 \end{cases}$

and $J_1^*\{L\} = (1+P_\theta)\gamma L + (1-\gamma)P_0 L(1-P_\theta)$.

iii) If $P_0 \le \gamma < P_1 - \dfrac{P_1 P_\theta}{1+P_1 P_\theta}$, $\begin{cases} \alpha_1^* = 1, & \text{if } Z_1 = 1 \\ \alpha_1^* = 0, & \text{if } Z_1 = 0 \\ \alpha_2^* = 1, & \text{if } Z_2 = 1 \\ \alpha_2^* = 0, & \text{if } Z_2 = 0 \end{cases}$

and $J_1^*\{L\} = \left(2P_\theta\gamma - 2P_\theta P_0 + 2P_0 - P_\theta P_0\gamma + P_\theta^2 P_0\gamma - P_0^2 - P_\theta^2 P_0^2 + 2P_\theta P_0^2\right)L$.

iv) If $P_1 - \dfrac{P_1 P_\theta}{1+P_1 P_\theta} \le \gamma < P_1$, $\begin{cases} \alpha_1^* = 0 \\ \alpha_2^* = 1, & \text{if } Z_2 = 1 \\ \alpha_2^* = 0, & \text{if } Z_2 = 0 \end{cases}$

and $J_1^*\{L\} = \left(P_\theta^2\left(P_1 - \gamma\right) + 2P_\theta + P_\theta\gamma - P_\theta^2 - P_\theta P_1\right)L$.

v) If $\gamma \ge P_1$, $\begin{cases} \alpha_1^* = 0 \\ \alpha_2^* = 0 \end{cases}$

and $J_1^*\{L\} = P_\theta L + (1-P_\theta)P_\theta L$.

---

[3] It can be noticed that if the cost of protection is very high, the farmer will not protect the harvest whatever the forecast probabilities may be. On the other hand, for low enough values the farmer will take the protective action whatever the forecast given. However there are intermediate regions where the decision clearly depends on the information quality.

(B) Let us assume that $P_0 > \frac{P_1}{1+P_1}$ *(low quality of the meteorological information).*
The optimal policy for the minimization problem in [2] is:

i) If $\gamma < \frac{P_0}{1+P_0}, \begin{cases} \alpha_1^* = 1 \\ \alpha_2^* = 1 \end{cases}$

and $J_1^*\{L\} = 2\gamma L.$

ii) If $\frac{P_0}{1+P_0} \leq \gamma < \frac{P_1}{1+P_1}, \begin{cases} \alpha_1^* = 1, \text{ if } Z_1 = 1 \\ \alpha_1^* = 0, \text{ if } Z_1 = 0 \\ \alpha_2^* = 1 \end{cases}$

and $J_1^*\{L\} = (1+P_\theta)\gamma L + P_0(1-\gamma)L(1-P_\theta).$

iii) If $\frac{P_1}{1+P_1} \leq \gamma < P_0, \begin{cases} \alpha_1^* = 0 \\ \alpha_2^* = 1 \end{cases}$

and $J_1^*\{L\} = P_\theta L + \gamma(1-P_\theta)L.$

iv) If $P_0 \leq \gamma < P_1, \begin{cases} \alpha_1^* = 0 \\ \alpha_2^* = 1, \text{ if } Z_2 = 1 \\ \alpha_2^* = 0, \text{ if } Z_2 = 0 \end{cases}$

and $J_1^*\{L\} = \left(P_\theta^2\left(P_1 - \gamma\right) + 2P_\theta + P_\theta\gamma - P_\theta^2 - P_\theta P_1\right)L.$

v) If $\gamma \geq P_1, \begin{cases} \alpha_1^* = 0 \\ \alpha_2^* = 0 \end{cases}$

and $J_1^*\{L\} = P_\theta L + (1-P_\theta)P_\theta L.$

The proof is in the Annex.

Comparison with the static case:

In the static case (one period), the optimal policy when meteorological information is available is the following: to protect if $\gamma < P_0$ to protect when $Z = 1$ but not to protect when $Z = 0$ if $P_0 \leq \gamma < P_1$, and not to protect if $\gamma \geq P_1$. Therefore, if the cost of protection is low it is better to protect, if that cost is high it is better not to protect, and if it takes an intermediate value the corresponding optimal decision has to be taken depending on the specific forecast.

Again, as in the case of only climatological information, the optimal policy obtained in the two-period case is less protective than that corresponding to the static case:

- When the condition for unconditional no protection in the static case holds $(\gamma \geq P_1)$, it is optimal not to protect in each of the two periods in the dynamic case.
- When the parameter γ takes an intermediate value ($P_0 \leq \gamma < P_1$) and the meteorological information is of high quality (case *A*), the optimal policy to apply is the one obtained in the static case in each of both periods if $P_0 \leq \gamma < P_1 - \frac{P_1 P_\theta}{1 + P_1 P_\theta}$, but the optimal policy is not to protect in the first period and to apply the optimal policy corresponding to the static case only in the second period is $P_1 - \frac{P_1 P_\theta}{1 + P_1 P_\theta} \leq \gamma < P_1$.
- When the parameter γ takes that intermediate value ($P_0 \leq \gamma < P_1$) but the meteorological information is of low quality (case *B*), the optimal policy is not to protect in the first period and to protect in the second period only if a forecast of adverse weather has been received for that period (that is, the optimal policy corresponding to the static case, only in the second period).

- The protective situation in the static model ($\gamma < P_0$) is divided in two cases in the dynamic model (*A*): to protect in each of the two periods if the cost of protection is very low $\left(\gamma < \frac{P_0}{1 + P_0}\right)$ and to protect in the second period but to protect in the first period only if a forecast of adverse weather has been received for that period.
- That protective situation in the static model ($\gamma < P_0$) is divided in three cases in the dynamic model (*B*): to protect in each of the two periods if the cost of protection is very low $\left(\gamma < \frac{P_0}{1 + P_0}\right)$, to protect in the second period but to protect in the first period only if a forecast of adverse weather has been received for that period if $\frac{P_0}{1 + P_0} \leq \gamma < \frac{P_1}{1 + P_1}$, and not to protect in the first period but to protect in the second period if $\frac{P_1}{1 + P_1} \leq \gamma < P_0$.

3.3. Economic value of the forecast information

The economic value of the forecast information is defined (Katz and Murphy, 1997) as:

V = Value of information = Minimum expected expense (wihtout forecasting) -
- Minimum expected expense (whith forecasting)

The minimum expected expense (without forecasting) corresponds to the value of $J_1^*\{L\}$ in Proposition 1, and depends on the parameters γ and P_θ.

The minimum expected expense (with forecasting) corresponds to the value of $J_1^*\{L\}$ in Proposition 2, and depends on the parameters γ, P_0, P_1 and P_θ. As in Section 2 it has been obtained that $P_0 = \frac{(1-P_1)P_\theta}{(1-P_\theta)}$, this optimal expense can be written as a function of the parameters γ, P_1 and P_θ.

Therefore, for γ and L given, the value of information can be written as a function of P_1 and P_θ. In order to write the value of information as a function of a unique variable, it is usual in the literature to consider $q = \frac{P_1 - P_\theta}{1-P_\theta} = Corr(\theta, Z)$, where $q \in [0,1]$ is what is called the quality of the information.

If we compare the structure of the optimal policies corresponding to the cases without forecasting (Proposition 1) and with forecasting (Proposition 2), we observe that there is an improvement in the decision, and therefore the economic value of the information is positive, in the case in which $\frac{P_0}{1+P_0} \le \gamma \le P_1$ holds (except when $\frac{P_1}{1+P_1} \le \gamma < P_0$, in the case of low quality of the forecast system), because in this interval the decision is dependent on the value predicted for Z, that is the forecast system result. Moreover in this case the value of $J_1^*\{L\}$ given by Proposition 1 is different from the corresponding value of $J_1^*\{L\}$ obtained in Proposition 2. In this situation, since the farmer's optimal decision depends on the meteorological information received, this information has a positive value in being able to improve the decision making process. This value will be derived from the difference between the optimal expected expense with climatological information and forecasting systems respectively available.

However, there are three situations in which the forecast information has no effect on the farmer's decision making: (i) When $\gamma > P_1$, the optimal decision is the same as in the case of climatological information, the farmer should not protect the harvest in either of the two time periods whatever the forecast received, and therefore the information value is zero in this case. The optimal expected expense given by Proposition 1 is identical to the corresponding expense obtained in Proposition 2: $J_1^*\{L\} = P_\theta L + (1-P_\theta)P_\theta L$. This does not depend on the quality of the information, so for this kind of situation improvements in the weather forecast systems will not revert in an increasing value of the information given. (ii) When $\gamma \le \frac{P_0}{1+P_0}$, the cost for the protection action is so low that the optimal decision will be to protect whatever the information received. The information value is therefore again zero, since the forecast does not change the decision of the farmer. In this case, $J_1^*\{L\} = 2\gamma L$, in both propositions 1 and 2. (iii) When $\frac{P_1}{1+P_1} \le \gamma < P_0$, in case *(B)* of low quality of the meteorological information. Observing the optimal policy when meteorological information is available when γ varies from 0 to 1 we see a transition from unconditional protection in both periods (for low values of γ) to different forms of conditional protection (for intermediate values of γ) to unconditional no protection in both periods (for high values of γ). In the case of low quality of the weather forecast system, if $\frac{P_1}{1+P_1} \leq \gamma < P_0$, it is optimal not to protect in the first period (instead of conditional protection) and to protect in the second period (Proposition 2, (B)). In this case, in Proposition 1 and Proposition 2 we have that $J_1^*\{L\} = P_\theta L + (1-P_\theta)P_\theta L$. Therefore in this particular case, the forecast is not used and the economic value of the information is zero because the difference between the minimum expected expense without forecasting and the corresponding expense with forecasting is zero.

4. A numerical example

In this section we obtain the optimal policy and the corresponding optimal value of the objective function for different values of the parameters. First we consider the case in which only climatological information is available and in the second subsection we study the case when weather or seasonal forecast information is also available.

4.1. Only climatological information is available

Let us assume that $L = 1$ (the amount of harvest available at the beginning of the time horizon is normalized to 1). As $C = \gamma L$ we have then that $C = \gamma$.

Table 2 summarizes the potential results for the two strategies available in each period.

TABLE 2

Expected expenditure for the different strategies in the two periods when no forecast system is available (P means Protect and N means Not to Protect)

Strategies		**Expected expenditure**		
Period 1	**Period 2**	**Period 1**	**Period 2**	**Total**
P	P	γ	γ	2γ
P	N	γ	P_θ	$\gamma + P_\theta$
N	P	P_θ	$(1 - P_\theta)\gamma$	$\gamma + P_\theta(1 - \gamma)$
N	N	P_θ	$(1 - P_\theta) P_\theta$	$(2 - P_\theta) P_\theta$

Source: Own work.

The optimal strategies and the corresponding optimal expected expenditures for different values of γ ($\gamma \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7\}$) and different values of P_θ ($P_\theta \in \{0.15, 0.30, 0.45, 0.60, 0.75\}$) are presented in Table 3.

TABLE 3

Optimal strategies (P: Protect, N: Not to Protect) and optimal expected expenditures (in brackets) for different values of the cost of protection (γ) and the probability of adverse weather (P_θ)

$\gamma \backslash P_\theta$	**0.15**	**0.30**	**0.45**	**0.60**	**0.75**
0.1	P,P (0.2)	P,P (0.2)	P,P (0.2)	P,P (0.2)	P,P (0.2)
0.2	N,N (0.277)	P,P (0.4)	P,P (0.4)	P,P (0.4)	P,P (0.4)
0.3	N,N (0.277)	N,N (0.51)	P,P (0.6)	P,P (0.6)	P,P (0.6)
0.4	N,N (0.277)	N,N (0.51)	N,P (0.67)	N,P (0.76)	P,P (0.8)
0.5	N,N (0.277)	N,N (0.51)	N,N (0.697)	N,P (0.8)	N,P (0.875)
0.6	N,N (0.277)	N,N (0.51)	N,N (0.697)	N,N (0.84)	N,P (0.9)
0.7	N,N (0.277)	N,N (0.51)	N,N (0.697)	N,N (0.84)	N,P (0.925)
0.8	N,N (0.277)	N,N (0.51)	N,N (0.697)	N,N (0.84)	N,N (0.937)

Source: Own work.

As can be seen in Table 3, for a fixed value of P_θ, the higher the cost of protection (higher value of γ), the lower the level of protection. On the other hand, for a fixed value of γ (a given cost of protection), the higher the probability of adverse weather, the higher the level of protection. As has been proved in Section 3, it is never optimal to protect in the first period and not to protect in the second period. In Table 3 we can also see that whenever the cost of protection (γ) is higher than or equal to the probability of adverse weather (P_θ), the optimal strategy is not to protect in any of the periods.

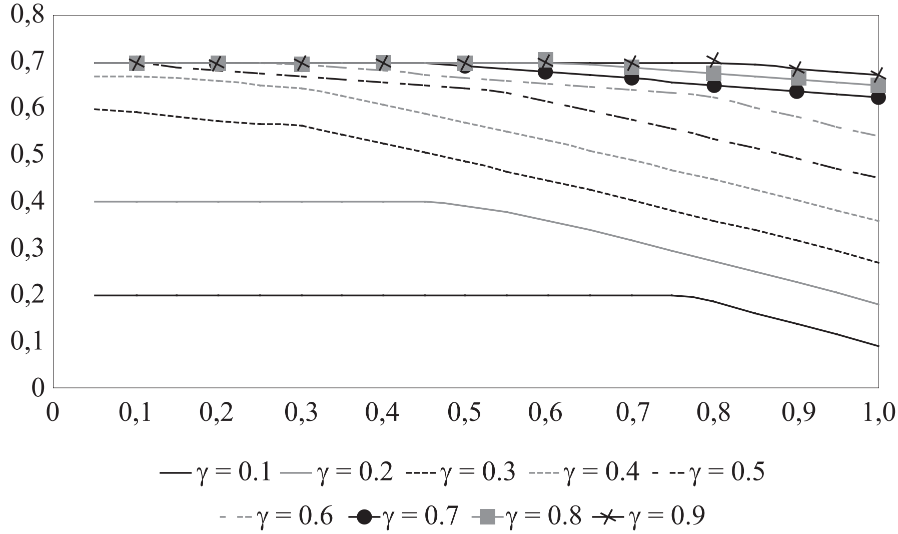
4.2. Meteorological information is available

In this subsection, the optimal expected expenses, when weather forecast information is available, are obtained, applying Proposition 2, for different values of the cost of protection, as a function of the quality of the information, for a given value of P_θ. Also, the corresponding economic values of the forecast information introduced in Section 3 are obtained.

Here, as in the previous case, it is assumed that $L = 1$. Therefore, we have that $C = \gamma$ is the cost of protection, with $0 < \gamma < 1$. All the numerical results are obtained for the case in which $P_\theta = 0.45$.

FIGURE 3

Optimal expected expenses for different values of *γ* as a function of *q* for $P_\theta = 0.45$ and $L = 1$



Source: Own work.

Figure 3 contains the optimal expected expenses for different values of the parameter *γ*, as a function of the quality of the information q, for $P_\theta = 0.45$. The optimal expected expenses are obtained for these particular numerical values of the parameters, applying Proposition 2, where each of the optimal expected expenses, in general also depends on P_0 and P_1.

As $P_0 = \frac{(1-P_1)P_\theta}{1-P_\theta}$ (see footnote 1), each of the optimal expected expenses, for these particular values of the parameters, can be written as a function of P_1. On the other hand, in Section 3 the quality of the information has been defined as $q = \frac{P_1 - P_\theta}{1-P_\theta}$. Therefore, for $P_\theta = 0.45$, we have that $P_1 = 0.55q + 0.45$, and the optimal expected expenses obtained in Proposition 2 can be expressed as a function of q, where $0 \leq q \leq 1$. The meaning of the variable q is more intuitive than of the variable P_1.

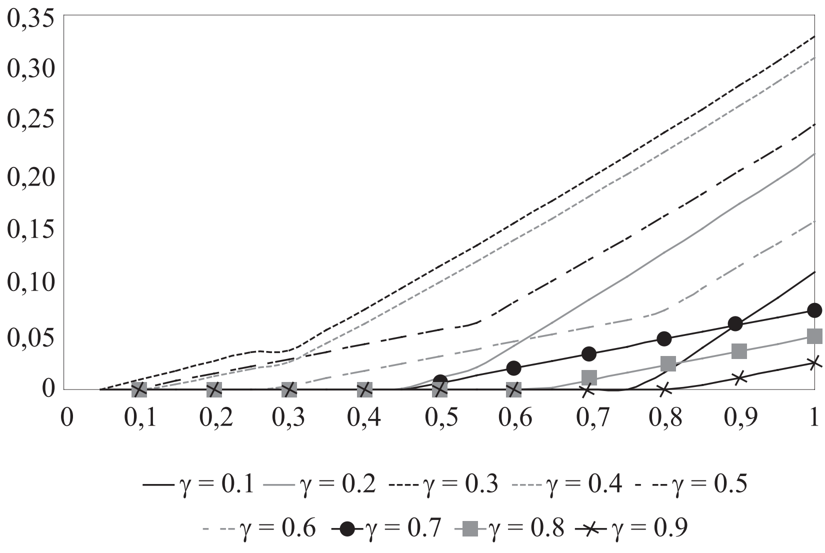
The following conclusions can be obtained from Figure 3:

- The higher the cost of protection (γ), the higher the optimal expected expense.
- For whatever γ fixed, the optimal expected expense decreases when the quality of the information increases.
- The rate of decrease of the optimal expected expense is higher when the quality of the information is higher.

Figure 4 contains the economic value of the forecast information $V(q)$, introduced in Section 3, for L=1 and $P_\theta = 0.45$, for different values of γ, as a function of the quality of the information q. For the calculation of $V(q)$, the optimal expected expenses obtained from Proposition 2 to represent Figure 3, and also the corresponding expenses, for the case of climatological information, obtained from Proposition 1, are needed.

FIGURE 4

Economic value of the forecast information for different values of γ as a function of q for $P_\theta = 0.45$ and L=1



Source: Own work.

The following conclusions can be obtained from Figure 4:

- For whatever γ given, the economic value of the forecast information $V_\gamma(q)$ is a non-decreasing function of the quality of the forecast information q.

- For whatever γ given, there exists q^*_γ with $0 < q^*_\gamma < 1$, such that the form of the function $V_\gamma(q)$ is the following:

$$V_\gamma(q) = \begin{cases} 0, & \text{for } 0 \leq q \leq q_\gamma^* \\ W_\gamma(q), & \text{for } q_\gamma^* \leq q \leq 1, \end{cases}$$

where $W_\gamma(q)$ is strictly increasing and also linear or piecewise linear.

- When the quality of the forecast information is very low, the forecast information has no economic value. When the quality of the forecast information is high, the forecast information has high economic value.
- Initially, for low values of the cost of protection γ, the economic value of the forecast information is higher when the cost of protection is higher, up to a certain level ($\gamma = 0.3$ in this case). Beyond that level, the economic value of the forecast information is lower when the cost of protection is higher.

5. Conclusions

This paper develops analytical solutions for a dynamic cost-loss problem considering farmer's decisions about protection against adverse weather events through two different periods. We prove that several regions exist in the optimal policy depending on the cost for crop protection, the avoided loss and the quality of the information available. The optimal expected expense has been calculated for each of the regions and the economic value of forecast systems has been discussed. This value is derived from the difference between the optimal expected expense with climatological information and forecasting systems respectively available. The information value depends on the information quality, being zero below a quality threshold and taking positive values from this threshold and the value increasing with the information quality q. However, there are three situations in which the forecast information has no effect on the farmer's decision making: (i) the cost for protection being relatively high with respect to the probability of suffering a potential loss, because the farmer will not protect the harvest in either of the two seasons whatever the forecast received, and therefore the information value is zero. (ii) In the case of the cost for the protection action being so low that the optimal decision will be to protect whatever the information received. (iii) When $\frac{P_1}{1+P_1} \leq \gamma < P_0$, in the case of low quality of the weather forecast system. For these three cases, the information value is therefore zero, since the forecast does not change the decision of the farmer. This does not depend on the quality of the information, so for this kind of situations, improvements for the weather or seasonal forecast systems will not revert in an increasing value of the information given. Finally, a numerical example is presented in order to illustrate all the previous results. The optimal decision of the farmer and the corresponding minimum expected expense are obtained for different specific values of the parameters. The case in which only climatological information is available and also the case in which weather or seasonal forecasts are also available are analized. In this last case, the economic value of the meteorological information is also studied. Interesting conclusions are obtained and presented for this numerical result, in Section 4.

6. References

Battisti, D.S. and Naylor, R.L. (2009). "Historical warnings of future food insecurity with unprecedented seasonal heat". *Science,* 323(5911): 240-244. http://doi.org/b3wq4p.

Brouwer, F., Heinz, I. and Zabel, T. (2004). *Governance of water-related conflicts in agriculture. New directions in Agri-environmental and water policies in the EU.* Kluwer Academic Publishers, The Netherlands.

Cerdá, E. and Quiroga, S. (2010). "Economic value of weather forecasting: the role of risk aversion". *TOP: An Official Journal of the Spanish Society of Statistics and Operations Research*, 19(1) : 130-149. http://doi.org/cdsr5p.

Cerdá, E. and Quiroga, S. (2011). "Optimal crop protection against climate risk in a dynamic cost-loss decision-making model". *International Journal of Ecological Economics and Statistics. Special Issue: Non-Strictly Economics Motivations And Intergenerational Equity In Ecological Models*, 23(F11): 129-149.

Ciscar, J.C., Iglesias, A., Feyen, L., Szabo, L., Van Regemorter, D., Amelung, B., Nicholls, R., Watkiss, P., Christensen, O.B., Dankers, R., Garrote, L., Goodess, C.M., Hunt, A., Moreno, A., Richards, J. and Soria, A. (2011). "Physical and economic consequences of climate change in Europe". *Proceedings of the National Academy of Sciences,* 108(7): 2678-2683. http://doi.org/fvsgkt.

EU COM. (2009). "White paper - Adapting to climate change: Towards a European framework for action". *European Commission*. Available at: http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:52009DC0147:EN:NOT.

Gómez-Limón, J.A. and Riesgo, L. (2004). "Irrigation water pricing: Differential impacts on irrigated farms". *Agricultural Economics*, 31(1): 47-66. http://doi.org/bv9tct.

Iglesias A., Cancelliere A., Cubillo F., Garrote L. and Wilhite, D.A. (2009). *Coping with drought risk in agriculture and water supply systems: Drought management and policy development in the Mediterranean.* Springer, The Netherlands.

IPCC. (2007). *Fourth Assessment Report of the Intergovernmental Panel on Climate Change.* Cambridge University Press, Cambridge.

Katz, R.W. (1993). "Dynamic cost-loss ratio decision making model with an autocorrelated climate variable". *Journal of Climate*, 6(1): 151-160. http://doi.org/frxgnj.

Katz, R.W. and Murphy, A.H. (1997). "Forecast value: Prototype decision - making models". In Katz, R.W. and Murphy, A.H. (Eds.): *Economic Value of Weather and Climate Forecasts* . Cambridge University Press, Cambridge: 183-217.

Katz, R.W. and Ehrendorfer, M. (2006). “Bayesian Approach to Decision Making Using Ensemble Weather Forecasts”. *Weather and Forecasting*, 21: 220-231. http://doi.org/cqhdwh.

Meza, F.J., Wilks, D.S., Rihab, S.J. and Stedingerc, J.R. (2003). “Value of perfect forecasts of sea surface temperature anomalies for selected rain-fed agricultural locations of Chile”. *Agricultural and Forest Meteorology*, 116(3-4): 7-135. http://doi.org/fbzq9t.

Murphy, A.H., Katz, R.W., Winkler, R.L. and Hsu, W. (1985). “Repetitive decision making and the value of forecasts in the cost-loss ratio situation: A dynamic model”. *Monthly Weather Review*, 113: 801-813. http://doi.org/bjsmf3.

Olesen, J. and Bindi, M. (2002). “Consequences of climate change for European agricultural productivity, land use and policy”. *European Journal of Agronomy*, 16(4): 239-262. http://doi.org/b552st.

Palmer, T.N. (2002). “The Economic Value of Ensemble forecasts as a tool for risk assessment: From days to decades”. *Quarterly Journal of Royal Meteorological Society,* 128(581): 747-774. http:// doi.org/cjtqnv.

Wilks, D.S. (1997). “Forecast value: prescriptive decision studies”. In Katz, R.W. and Murphy, A.H. (Eds.): *Economic Value of Weather and Climate Forecasts.* Cambridge University Press, Cambridge: 109-145.

Annex: Proofs

Proof of Proposition 1:

Using the Dynamic Programming method for two periods, we start the analysis when the second period has finished.

End of the time horizon: L_3 being determined, with no contribution to the objective function: $J_3^* \{L_3\} = 0$.

Second period:

Second period (with L_2 determined): the Bellman equation corresponding to this period is: $J_2^*\{L_2\} = \underset{\alpha_2 \in \{0,1\}}{Min} E\{\gamma\alpha_2 L_2 + (1-\alpha_2)\theta_2 L_2\}$, which calculating the expected value: $E\{\gamma\alpha_2 L_2 + (1-\alpha_2)\theta_2 L_2\} = \gamma\alpha_2 L_2 + (1-\alpha_2) P_\theta L_2$, becomes: $J_2^*\{L_2\} = Min\{\gamma L_2, P_\theta L_2\}$.

Since L_2 can take the values 0 or L, we have two possibilities:

$$J_2^* \{0\} = 0 \text{ or } J_2^* \{L\} = Min \{\gamma L, P_\theta L\}.$$

So the contingent decision for the second period would be: α_2 (0): Whatever

(The decision in the second period is nonsense since in this case the crop was lost in the first period).

$$\alpha_2(L) = \begin{cases} 1, & \text{if } \gamma L < P_\theta L \Leftrightarrow \gamma < P_\theta \\ 0, & \text{if } \gamma L \geq P_\theta L \Leftrightarrow \gamma \geq P_\theta \end{cases}$$

That is, if the crop has suffered no damage by the beginning of the second period, the optimal decision depends on the cost-loss ratio. If the cost-loss ratio is higher (lower) than the probability of suffering the loss, then the better option is not to protect (protect). This decision should be the same if the farmer has to decide in just one period (the decision being static). If the crop was lost in the first period, the decision in the second period is irrelevant.

First period:

L_1 determined and equal to L.

The Bellman equation corresponding to the first period will be:

$$J_1^*\{L\} = \underset{\alpha_1\in\{0,1\}}{Min}\, E\{\gamma\alpha_1 L + (1-\alpha_1)\theta_1 L + J_2^*\{L-(1-\alpha_1)\theta_1 L\}\},$$

which considering the expected value will become:

$$J_1^*\{L\} = \underset{\alpha_1\in\{0,1\}}{Min}\{\gamma\alpha_1 L + (1-\alpha_1)P_\theta L + P_\theta J_2^*\{\alpha_1 L\} + (1-P_\theta)J_2^*\{L\}\}.$$

Therefore:
If $\alpha_1 = 0$, the expected value will be $P_\theta L + (1-P_\theta)\, Min\, \{\gamma L, P_\theta L\}$.
If $\alpha_1 = 1$, the expected value will be $\gamma L + Min\, \{\gamma L, P_\theta L\}$.

So, the farmer should protect in the first period if:

$$P_\theta L + (1-P_\theta)\, Min\, \{\gamma L, P_\theta L\} > \gamma L + Min\, \{\gamma L, P_\theta L\} \Leftrightarrow$$
$$P_\theta L - P_\theta\, Min\, \{\gamma L, P_\theta L\} > \gamma L \Leftrightarrow P_\theta - P_\theta\, Min\, \{\gamma, P_\theta L\} > \gamma$$

If $\gamma < P_\theta$ to protect will be optimal if $P_\theta - P_\theta\gamma > \gamma \Leftrightarrow \gamma < \frac{P_\theta}{P_\theta+1}$.

Therefore, in this case the farmer will protect in both seasons, $\alpha_1 = 1$ and $\alpha_2 = 1$, if $\gamma < \frac{P_\theta}{P_\theta+1} < P_\theta$, the expected expense is $J_1^*\{L\} = 2\gamma L$.

And it will protect just in the second season, that is $\alpha_1 = 0$ and $\alpha_2 = 1$ if $\frac{P_\theta}{P_\theta+1} \leq \gamma < P_\theta$. In that case, $J_1^*\{L\} = P_\theta L + \gamma(1 - P_\theta)L$.

It will never be optimal to protect in the first season and not to protect in the second, that is, $\alpha_1 = 1$ and $\alpha_2 = 0$, since it is impossible to satisfy $\gamma < \frac{P_\theta}{P_\theta+1}$ and $\gamma \geq P_\theta$ simultaneously.

If $\gamma \geq P_\theta$, to protect will be optimal if $P_\theta - P_\theta^2 > \gamma$, which is impossible, because $P_\theta - P_\theta^2 = P_\theta(1 - P_\theta) < P_\theta \leq \gamma$.

So, in this case, the farmer will not protect either in the first season or in the second, $\alpha_1 = 0$ and $\alpha_2 = 0$, and the expected expense will be $J_1^*\{L\} = P_\theta L + (1 - P_\theta)P_\theta L$.

Proof of Proposition 2:

Using the Dynamic Programming method for two periods, we start the analysis when the second period has finished.

End of the time horizon: L_3 being determined, with no contribution to the objective function: $J_3^* \{L\} = 0$.

Second period:

Second period (with L_2 and Z_2 determined): the Bellman equation corresponding to this period is: $J_2^* \{L_2, Z_2\} = \underset{\alpha_2 \in \{0,1\}}{Min} \underset{\theta_2}{E} \left\{ \left[\gamma\alpha_2 L_2 + (1-\alpha_2)\theta_2 L_2 \right] / Z_2 \right\}$. The following possibilities arise:

If $Z_2 = 1$, taking the expected value, we obtain

$$J_2^* \{L_2, 1\} = \underset{\alpha_2 \in \{0,1\}}{Min} \left[\gamma\alpha_2 L_2 + P_1 (1-\alpha_2) L_2 \right],$$

so $\alpha^*_2 = 1 \Leftrightarrow \gamma L_2 \leq P_1 L_2 \Leftrightarrow \gamma \leq P_1$.

Therefore:

- If $\gamma < P_1 \Rightarrow \alpha^*_2 = 1$ and $J_2^* \{L_2,1\} = \gamma L_2$.
- If $\gamma > P_1 \Rightarrow \alpha^*_2 = 0$ and $J_2^* \{L_2,1\} = P_1 L_2$.
- If $\gamma = P_1$, both options are indifferent for the farmer.

In general, $J_2^* \{L_2,1\} = Min\{ \gamma L_2, P_1 L_2\}$.

If $Z_2 = 0$, taking the expected value, we obtain:

$$J_2^* \{L_2, 0\} = \underset{\alpha_2 \in \{0,1\}}{Min} \left[\gamma\alpha_2 L_2 + P_0 (1-\alpha_2) L_2 \right]$$

so $\alpha^*_2 = 1 \Leftrightarrow \gamma L_2 \leq P_0 L_2 \Leftrightarrow \gamma \leq P_0$.

Therefore,

- If $\gamma < P_0 \Rightarrow \alpha^*_2 = 1$ and $J_2^* \{L_2,0\} = \gamma L_2$.
- If $\gamma > P_0 \Rightarrow \alpha^*_2 = 0$ and $J_2^* \{L_2,0\} = P_0 L_2$.
- If $\gamma = P_0$, both options are indifferent for the farmer.

In general, $J_2^* \{L_2,1\} = Min \{\gamma L_2, P_0 L_2\}$.

Now if we consider L_2 given but Z_2 still not known, the optimal value can be expressed by:

$$J_2^* \{L_2\} = \underset{Z_2}{E} \{J_2^* (L_2, Z_2)\} = \Pr \{Z_2 = 1\}\, J_2^* \{L_2, 1\} + \Pr \{Z_2 = 0\}\, J_2^* \{L_2, 0\} =$$
$$= P_\theta Min\{\gamma L_2, P_1 L_2\} + (1 - P_\theta)\, Min \{\gamma L_2, P_0 L_2\}.$$

And the optimal contingent plan will be:

- If $\gamma < P_0$, then $\alpha^*_2 = 1$, whatever the value predicted for Z_2.
- If $P_0 \leq \gamma < P_1$, then $\begin{cases} \alpha_2^* = 1, & \text{if } Z_2 = 1 \\ \alpha_2^* = 0, & \text{if } Z_2 = 0 \end{cases}$.

- If $\gamma \geq P_1$, then $\alpha^*_2 = 0$, whatever the value predicted for Z_2.

Therefore:

$$J_2^*\{L_2\} = \begin{cases} P_\theta \gamma L_2 + (1-P_\theta)\gamma L_2 = \gamma L_2, & \text{if} \quad \gamma < P_0 \\ P_\theta \gamma L_2 + (1-P_\theta) P_0 L_2, & \text{if} \quad P_0 \leq \gamma < P_1 \\ P_\theta P_1 L_2 + (1-P_\theta) P_0 L_2, & \text{if} \quad \gamma \geq P_1 \end{cases}$$

Since L_2 can take the values *0* or L, we have that:

$\alpha^*_2(0)$: Whatever (the decision in the second period is nonsense since in this case the crop was lost in the first period). Moreover, $J^*_2\{0\} = 0$.

$$J^*_2\{L, 1\} = Min\ \{\gamma L, P_1 L\}$$
$$J^*_2\{L, 0\} = Min\ \{\gamma L, P_0 L\}$$

$$\alpha_2^*(L) = \begin{cases} 1, \forall Z_2 & \text{when } \gamma < P_0 \\ 1 \text{ if } Z_2 = 1 \text{ and } 0 \text{ if } Z_2 = 0 & \text{when } P_0 \leq \gamma < P_1 \\ 0, \forall Z_2 & \text{when } \gamma \geq P_1 \end{cases}$$

$$\text{Moreover, } J_2^*\{L\} = \begin{cases} \gamma L, & \text{if} \quad \gamma < P_0 \\ P_\theta \gamma L + (1-P_\theta) P_0 L, & \text{if} \quad P_0 \leq \gamma < P_1 \\ P_\theta P_1 L + (1-P_\theta) P_0 L, & \text{if} \quad \gamma \geq P_1 \end{cases}$$

That is also the optimal policy for the static case (just one period).

First period:

With $L_1 = L$ and Z_1 known, the Bellman equation corresponding to the first period will be

$$J_1^*\{L, Z_1\} = \underset{\alpha_1 \in \{0,1\}}{Min} \ \underset{\theta_1}{E}\left\{\gamma \alpha_1 L + (1-\alpha_1)\theta_1 L + J_2^*\left\{\left[1-\theta_1(1-\alpha_1)\right]L\right\} / Z_1\right\}$$

We solve the problem for the two possible values of the forecast $Z_1 = 1$ and $Z_1 = 0$.

If $Z_1 = 1$, taking the expected value, we obtain

$$J_1^*\{L, 1\} = \underset{\alpha_1 \in \{0,1\}}{Min} \ \left\{\gamma \alpha_1 L + (1-\alpha_1) P_1 L + J_2^*\left\{\left[1-P_1(1-\alpha_1)\right]L\right\}\right\}$$

Substituting the value for α_1 we have the following possibilities for the objective function:

- If $\alpha_1 = 1$, the objective function is: $\gamma L + J_2^* \{L\}$.
- If $\alpha_1 = 0$, the objective function is: $P_1 L + J_2^* \{(1 - P_1)L\}$.

Since in the second period we obtained three possible optimal options depending on the value of γ, in order to determine the joint optimal policy for the two considered periods we have to analyse for the first period each of the three possibilities which appeared in the second period:

a) In the case of $\gamma < P_0$, we have:

If $\alpha_1 = 1$, the objective function is: $\gamma L + \gamma L = 2\gamma L$.

If $\alpha_1 = 0$, the objective function is: $P_1 L + \gamma(1 - P_1)L = P_1 L + \gamma L - \gamma P_1 L$.

Therefore the optimal decision will be $\alpha^*_1 = 0 \Leftrightarrow 2\gamma L \geq P_1 L + \gamma L - \gamma P_1 L \Leftrightarrow$

$\gamma L + \gamma P_1 L \geq P_1 L \Leftrightarrow \gamma + \gamma P_1 \geq P_1 \Leftrightarrow \gamma (1 + P_1) \geq P_1 \Leftrightarrow \gamma \geq \dfrac{P_1}{(1+P_1)}$.

- If $\gamma < P_0$ and $\gamma < \dfrac{P_1}{(1+P_1)} \Rightarrow \alpha_1^* = 1$ and $J_1^*\{L,1\} = 2\gamma L$.
- If $\gamma < P_0$ and $\gamma \geq \dfrac{P_1}{(1+P_1)} \Rightarrow \alpha_1^* = 0$ and $J_1^*\{L,1\} = P_1 L + \gamma L - \gamma P_1 L$.

In general, $J_1^* \{L, 1\} = Min\ \{2\gamma L, P_1 L + \gamma L - \gamma P_1 L\}$.

Therefore, distinguishing between the two cases considered in the statement of the Proposition, we have:

- Assuming that $P_0 \leq \dfrac{P_1}{1+P_1}$ (case *A*), if $\gamma < P_0$ it is $\alpha_1^* = 1$ and $J_1^*\{L,1\} = 2\gamma L$.
- Assuming that $P_0 > \dfrac{P_1}{1+P_1}$ (case *B*),

  if $\gamma < \dfrac{P_1}{1+P_1}$ it is $\alpha_1^* = 1$ and $J_1^*\{L,1\} = 2\gamma L$,

  if $\dfrac{P_1}{1+P_1} \leq \gamma < P_0$ it is $\alpha_1^* = 0$ and $J_1^*\{L,1\} = P_1 L + \gamma L - \gamma P_1 L$

b) In the case of $P_0 \leq \gamma < P_1$, we have:

If $\alpha_1 = 1$, the objective function is: $\gamma L + P_\theta \gamma L + (1 - P_\theta) P_0 L$

If $\alpha_1 = 0$, the objective function is: $P_1 L + P_\theta \gamma (1 - P_1) L + (1 - P_\theta) P_\theta (1 - P_1) L$,

And therefore, the optimal decision will be:

$$\alpha^*_1 = 1 \Leftrightarrow \gamma L + P_\theta \gamma L + (1 - P_\theta) P_0 L \leq P_1 L + P_\theta \gamma (1 - P_1) L + (1 - P_\theta) P_0 (1 - P_1) L \Leftrightarrow$$

$$\gamma \leq P_1 - P_1 P_\theta \gamma - P_1 P_0 + P_1 P_\theta P_0 \Leftrightarrow \gamma (1 + P_1 P_\theta) \leq P_1 - P_1 P_0 + P_1 P_\theta P_0 \Leftrightarrow$$

$$\gamma \leq \frac{P_1 - P_1 P_0 + P_1 P_\theta P_0}{\left(1 + P_1 P_\theta\right)} = P_1 - \frac{P_1 P_\theta}{1 + P_1 P_\theta}$$ [4]

- If $P_0 \leq \gamma < P_1$ and $\gamma \leq P_1 - \dfrac{P_1 P_\theta}{1 + P_1 P_\theta} \Rightarrow \alpha_1^* = 1$ and
  $J_1^* \{L,1\} = \gamma L + P_\theta \gamma L + \left(1 - P_\theta\right) P_0 L$
- If $P_0 \leq \gamma < P_1$ and $\gamma > P_1 - \dfrac{P_1 P_\theta}{1 + P_1 P_\theta} \Rightarrow \alpha_1^* = 0$ and
  $J_1^* \{L,1\} = P_1 L + P_\theta \gamma \left(1 - P_1\right) L + \left(1 - P_\theta\right) P_0 \left(1 - P_1\right) L$

In general for this case, we obtain:

$$J_1^* \{L,1\} = Min\left\{\gamma L + P_\theta \gamma L + \left(1 - P_\theta\right) P_0 L, P_1 L + P_\theta \gamma \left(1 - P_1\right) L + \left(1 - P_\theta\right) P_0 \left(1 - P_1\right) L\right\}.$$

As $P_0 \leq P_1 - \dfrac{P_1 P_\theta}{1 + P_1 P_\theta} = \dfrac{P_1 - P_1 P_0 + P_1 P_\theta P_0}{1 + P_1 P_\theta} \Leftrightarrow P_0 + P_0 P_\theta P_1 \leq P_1 - P_1 P_0 + P_0 P_\theta P_1 \Leftrightarrow$

$\Leftrightarrow P_0 + P_0 P_1 \leq P_1 \Leftrightarrow P_0 \leq \dfrac{P_1}{1 + P_1}$,

we can distinguish between the two cases considered in the statement of Proposition 2, in the following way:

- Assuming that $P_0 \leq \dfrac{P_1}{1 + P_1}$ (case *A*),

$$\begin{cases} \text{if } P_0 \leq \gamma < P_1 \text{-} \dfrac{P_1 P_\theta}{1 + P_1 P_\theta} \text{ it is } \alpha_1^* = 1 \text{ and } J_1^* \{L,1\} = \gamma L + P_\theta \gamma L + (1 - P_\theta) P_0 L, \\ \text{if } P_1 \text{-} \dfrac{P_1 P_\theta}{1 + P_1 P_\theta} \leq \gamma < P_1 \text{ it is } \alpha_1^* = 0 \text{ and } J_1^* \{L,1\} = P_1 L + P_\theta \gamma (1 - P_1) L + (1 - P_\theta) P_0 (1 - P_1) L \end{cases}$$

- Assuming that $P_0 > \dfrac{P_1}{1 + P_1}$ (case *B*),

if $P_0 \leq \gamma < P_1$ it is $\alpha_1^* = 0$ and $J_1^* \{L,1\} = P_1 L + P_\theta \gamma \left(1 - P_1\right) L + \left(1 - P_\theta\right) P_0 \left(1 - P_1\right) L.$

c) In the case of $\gamma \geq P_1$, we have:

If $\alpha_1 = 1$, the objective function is: $\gamma L + P_\theta P_1 L + \left(1 - P_\theta\right) P_0 L$

If $\alpha_1 = 1$, the objective function is: $P_1 L + P_\theta P_1 \left(1 - P_1\right) L + \left(1 - P_\theta\right) P_0 \left(1 - P_1\right) L,$
and the optimal policy will be:

[4] It is $0 < P_1 - \dfrac{P_1 P_\theta}{1 + P_1 P_\theta} < P_1$, because $\dfrac{P_1 P_\theta}{1 + P_1 P_\theta} < P_1 \Leftrightarrow P_\theta < 1 + P_1 P_\theta$ which is evident, and $\dfrac{P_1 P_\theta}{1 + P_1 P_\theta} > 0.$

$$\alpha_1^* = 1 \Leftrightarrow \gamma L + P_\theta P_1 L + (1-P_\theta) P_0 L \le P_1 L + P_\theta P_1 (1-P_1) L + (1-P_\theta) P_0 (1-P_1) L \Leftrightarrow$$
$$\Leftrightarrow \gamma \le P_1 - P_\theta P_1^2 - P_0 P_1 (1-P_\theta) < P_1,$$

but this cannot occur if $\gamma \ge P_1$, therefore in both cases *A* and *B,*

- If $\gamma \ge P_1$ it is $\alpha^*_1 = 0$ and $J_1^* \{L,1\} = P_1 L + P_\theta P_1 (1-P_1) L + (1-P_\theta) P_0 (1-P_1) L.$

If $Z_1 = 0$, taking the expected value we obtain the following expression for the optimal expense:

$$J_1^* \{L,0\} = \underset{\alpha_1 \in \{0,1\}}{Min} \left\{ \gamma \alpha_1 L + (1-\alpha_1) P_0 L + J_2^* \left\{ \left[1 - P_0 (1-\alpha_1) \right] L \right\} \right\},$$

and substituting the value for α_1 we have the following possibilities for the objective function:

- If $\alpha_1 = 1$, the objective function is: $\gamma L + J_2^* \{L\}$.
- If $\alpha_1 = 0$, the objective function is: $P_0 L + J_2^* \{(1 - P_0)L\}$.

Again we consider the three possible options in the second period:

a) In the case of $\gamma < P_0$, we have:

- If $\alpha_1 = 1$, the objective function is: $\gamma L + \gamma L = 2 \gamma L$.
- If $\alpha_1 = 0$, the objective function is: $P_0 L + \gamma(1 - P_0)L = P_0 L + \gamma L - \gamma P_0 L$.

So, the optimal policy will be:

$$\alpha_1^* = 0 \Leftrightarrow 2\gamma L \ge P_0 L + \gamma L - \gamma P_0 L \Leftrightarrow \gamma (1+P_0) \ge P_0 \Leftrightarrow \gamma \ge \frac{P_0}{(1+P_0)}.$$

If $\gamma < P_0$ and $\gamma < \frac{P_0}{(1+P_0)} \Leftrightarrow \gamma < \frac{P_0}{(1+P_0)}$, then $\alpha_1^* = 1$ and $J_1^* \{L,0\} = 2\gamma L$

If $\gamma < P_0$ and $\gamma \ge \frac{P_0}{(1+P_0)} \Leftrightarrow \frac{P_0}{(1+P_0)} \le \gamma < P_0$,

then $\alpha_1^* = 0$ and $J_1^* \{L,0\} = P_0 L + \gamma L - \gamma P_0 L.$

In general, $J_1^* \{L,0\} = Min\{2\gamma L, P_0 L + \gamma L - \gamma P_0 L\}$.

Therefore, for both cases *(A)* and *(B)* we have,

$$\begin{cases} \text{if } \gamma < \dfrac{P_0}{1+P_0} \text{ it is } \alpha_1^* = 1 \text{ and } J_1^* \{L,0\} = 2\gamma L, \\ \text{if } \dfrac{P_0}{1+P_0} \le \gamma < P_0 \text{ it is } \alpha_1^* = 0 \text{ and } J_1^* \{L,0\} = P_0 L + \gamma L - \gamma P_0 L \end{cases}$$

b) In the case of $P_0 \leq \gamma < P_1$, we have:

- If $\alpha_1 = 1$, the objective function is: $\gamma L + P_\theta \gamma L + (1 - P_\theta)P_0 L$.
- If $\alpha_1 = 0$, the objective function is: $P_0 L + P_\theta \gamma (1 - P_0)L + (1 - P_\theta) P_0(1 - P_0)$L.

So, the optimal policy will be:

$$\alpha_1^* = 1 \Leftrightarrow \gamma L + P_\theta \gamma L + \left(1 - P_\theta\right) P_0 L \leq P_0 L + P_\theta \gamma \left(1 - P_0\right) L + \left(1 - P_\theta\right) P_0 \left(1 - P_0\right) L \Leftrightarrow$$
$$\Leftrightarrow \gamma \leq P_0 - \frac{P_0^2}{1 + P_\theta P_0} < P_0,$$

but it is impossible since $\gamma \geq P_0$.

Therefore, in both cases *(A)* and *(B):*

- If $P_0 \leq \gamma < P_1$ and it is $\alpha^*_1 = 0$ and
$J_1^*\{L, 0\} = P_0 L + P_\theta \gamma \left(1 - P_0\right) L + \left(1 - P_\theta\right) P_0 \left(1 - P_0\right) L$

c) In the case of $\gamma \geq P_1$, we have:

- If $\alpha_1 = 1$, the objective function is: $\gamma L + P_\theta P_1 L + (1 - P_\theta)P_0 L$.
- If $\alpha_1 = 0$, the objective function is: $P_0 L + P_\theta P_1 (1 - P_0)L + (1 - P_\theta) P_0(1 - P_0)$L.

so the optimal policy will be:

$$\alpha_1^* = 1 \Leftrightarrow \gamma L + P_\theta \gamma L + \left(1 - P_\theta\right) P_0 L \leq P_0 L + P_\theta \gamma \left(1 - P_0\right) L + \left(1 - P_\theta\right) P_0 \left(1 - P_0\right) L \Leftrightarrow$$
$$\Leftrightarrow \gamma \leq P_0 - \frac{P_0^2}{1 + P_\theta P_0} < P_0,$$

which is impossible since $\gamma \geq P_1 \geq P_0$.

Therefore, for both cases *(A)* and *(B),*

If $\gamma \geq P_1 \;\Rightarrow \alpha_1^* = 0$ and $J_1^*\{L, 0\} = P_0 L + P_\theta P_1 \left(1 - P_0\right) L + \left(1 - P_\theta\right) P_0 \left(1 - P_0\right) L$.

To calculate the a-priori expected expense, which is the expected result before even knowing what the prevision is for the first period, we consider $L_1 = L$ given and Z_1 random variable. The optimal expected expense can be calculated as:

$$J_1^*\{L\} = \underset{Z_1}{E}\left\{J_1^*\left(L, Z_1\right)\right\} = \Pr\{Z_1 = 1\} J_1^*\{L, 1\} + \Pr\{Z_1 = 0\} J_1^*\{L, 0\} =$$
$$= P_\theta J_1^*\{L, 1\} + (1 - P_\theta) J_1^*\{L, 0\}.$$

Substituting these expressions in each case and each interval, the optimal policy and the corresponding optimal value of the objective function is obtained.