System Times and Channel Availability for Secondary Transmissions in CRNs: A Dependability Theory based Analysis

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***Abstract*—Reliability is of fundamental importance for the performance of secondary networks in cognitive radio networks (CRNs). To date, most studies have focused on predicting reliability parameters based on prior statistics of traffic patterns from user behavior. In this paper, we define a few reliability metrics for channel access in multi-channel CRNs which are analogous to the concepts of reliability and availability in the classical dependability theory. Continuous time Markov chains (CTMC) are employed to model channel available and unavailable time intervals based on channel occupancy status. The impact on user access opportunities based on channel availability is investigated by analyzing the steady state channel availability and several system times such as mean channel available time and mean time to first channel unavailability. Moreover, the complementary cumulative distribution function for channel availability is derived by applying the uniformization method and it is evaluated as a measure of guaranteed availability for channel access by secondary users. The preciseness and the correctness of the derived analytical models are validated through discrete-event based simulations. We believe that the reliability metric definitions and the analytical models proposed in this paper have their significance for reliability and availability analysis in CRNs.**

***Index Terms*—Cognitive radio networks, spectrum access, channel availability, system times, guaranteed availability, CTMC**

I. Introduction

During the past decade, the research interests on cognitive radio networks (CRNs) have been growing tremendously due to CRN's capability of exploiting the unused spectrum in dynamically changing environments [1], [2]. The cognition and reconfigurability features of cognitive radios (CRs) tackle the problem of spectrum scarcity, improving the quality of service (QoS) as well as communication reliability. Dynamic spectrum access is one of the key features for CRNs which allows secondary users (SUs) occupy unused channels in an opportunistic manner. In this way, primary users (PUs) who own the license have priority for spectrum access and SUs can also access the channel when the spectrum is not occupied by PUs. Consequently, the system performance of the secondary network (SN) depends on the dynamics of the primary network.

In order to improve the performance of SNs, various techniques have been proposed in the literature. The focus of these approaches has been on techniques including spectrum sensing [3], channel access schemes and resource allocation [4], [5], spectrum management [6], etc. However, little attention has been paid so far to the reliability and availability performance of SU networks from the perspective of dependability theory. Indeed, *dependability* is one of the four major performance attributes[1] for any computing and communication systems since it has direct impact on the performance of the system [7]. Therefore, studying the reliability and availability aspects of CRNs is of essential importance for real-life deployment of CRNs in the near future. This paper makes an effort towards this direction by defining and investigating several reliability and availability metrics for channel access in multi-channel CRNs based on dependability theory.

The dependability of a computing or communication system can be described as its ability to deliver services that can justifiably be trusted [8]. Therein, reliability, availability, safety and confidentiality are the main attributes which describe the properties of a system with respect to dependability. In a CRN, PUs always have priority for channel access while SUs can only opportunistically access the idle channels. Once all the channels are occupied at a given instant, a new secondary request will simply be blocked and the CRN is said to be unavailable for new SUs. Thus, channel availability is an important metric for SNs to be considered when accessing channels in a CRN. Furthermore, the analysis of channel idleness from the *dependability perspective* provides a systematic approach to evaluate the time intervals related to channel availability and unavailability periods.

The importance of employing *dependability theory* for reliability analysis is twofold. First and most importantly, the terminologies related to QoS of telecommunication services have been defined in the *ITU-T E.800 Recommendation* [9] based on the dependability theory. Correspondingly, our definitions of reliability metrics introduced in this study conform with

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[1]In general, computing and communication systems are characterized by four main properties: functionality, performance, cost and dependability.

the ITU recommendations since dependability theory forms the basis for our study. Second, the four types of system times (mean time to first channel unavailability, mean time to channel unavailability, mean channel available time and mean channel unavailable time) defined in this study can be adopted for accurate and systematic reliability analysis in CRNs from the perspective of the dependability theory rather than using other empirical rules or conventional performance metrics like packet error probability or packet delivery ratio. Therefore the validity and the eligibility of the definitions for reliability measures are confirmed and the model developed in this study provides a systematic approach for reliability analysis in CRNs and it is applicable to realistic scenarios.

In this paper, we focus particularly on two categories of dependability measures, i.e., availability and system times, for spectrum access in CRNs. The goal of this study is to define those dependability measures which constitute the basis for determining channel availability in CRNs and to develop analytical models to calculate system availability and its distribution. In brief, the main contributions of this paper are summarized as follows:

- A number of reliability and availability metrics for channel access in CRNs are defined from *the perspective of dependability theory*, including mean time to first channel unavailability, mean time to channel unavailability, mean channel available and unavailable times, and steady state channel availability.
- A Markov process based approach is proposed to calculate channel availability as well as related system times in a multi-channel CRN. In addition to channel availability, we introduce a scale of guaranteed availability and deduce the complementary cumulative distribution function (CCDF) of channel availability of CRNs.
- The proposed model is evaluated by adopting an existing channel access scheme from our early work [4] to investigate how the reliability of a CRN is influenced by traffic load under both homogeneous and heterogeneous channel conditions. Furthermore, the dependability measures of the SN are evaluated using a queuing scheme as an extension to the existing scheme. The proposed model is however independent of the channel access schemes employed.
- The correctness and the preciseness of the derived analytical models are validated through discrete-event based simulations. Additionally, channel availability under lognormal PU interarrival time and SU service time distributions is also validated by means of simulations in order to assess the applicability of our model under non-exponentially distributed interarrival and service times.

The remainder of this paper is organized as follows. In Sec. II, we provide background information on the concept of reliability in communication systems and translate it into CRNs. The related work is also summarized therein. Sec. III outlines the network scenario and the dynamic channel access scheme employed in this study. Then the procedure of the proposed analytical model to calculate channel availability and system times in a CRN is presented in Sec. IV, followed by the detailed description of the numerical approach to determine the CCDF of channel availability in Sec. V. In Sec. VI, the numerical results are presented and discussed. Finally, the conclusions are drawn in Sec. VII.

II. Reliability and Availability in CRNs: Background and Related Work

In this section, we first revisit the classical reliability concepts in communication networks and present briefly the concepts of reliability in CRNs introduced in this study. Afterward, the related work is summarized.

A. Background Information on Reliability Metrics

In communications networks and electronic systems, the time to first failure is the time from the system initiation instant until it fails for the first time. For *non-repairable* systems or components like integrated circuits, the mean time to first failure (MTFF) is a critical dependability parameter which indicates the system's expected lifetime. Denote a random variable which represents the time to first failure as TFF and the cumulative distribution function of time to first failure as $F_{TFF}(t)$. We have

$$F_{\text{TFF}}(t) = P(\text{TFF} \leq t) = \int_0^t f_{\text{TFF}}(x)\,dx \quad \text{for} \quad t \geq 0, \quad (1)$$

where $f_{\text{TFF}}(t)$ is the probability density function of the random variable TFF. The reliability function $R(t)$ is defined as the probability that a system can provide its required services under stated conditions for a given time interval. Therefore,

$$R(t) = P(\text{TFF} > t) = 1 - F_{\text{TFF}}(t) = \int_t^\infty f_{\text{TFF}}(x)\,dx. \quad (2)$$

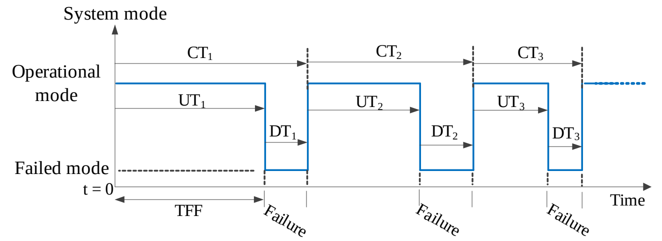
In *repairable* systems, the mean time to failure (MTTF) is another parameter indicating the time from a *random instant* when the system is working and is in its steady state until it fails. The available time or uptime is the time during which the system is operational. The unavailable time or downtime is the time during which the system is not operational. Observe the system for a cycle which includes a period of uptime and a period of downtime respectively. Then the mean cycle time, MCT $=$ MUT $+$ MDT, where MUT and MDT represent mean uptime and mean downtime respectively. Correspondingly, the steady state availability, A_{ss}, is obtained as the ratio between the *long run average* of the uptime and the cycle time, expressed as,

$$A_{ss} = \frac{\text{MUT}}{\text{MUT} + \text{MDT}}. \quad (3)$$

B. The Concepts of Reliability in CRNs

In this subsection, we initiate the concepts of reliability in CRNs. Later on in Subsec. IV-C, we will give the definitions of channel availability and the related system times mathematically. To do so, the above mentioned reliability metrics are revised such that they can be applied in multi-channel CRNs without deviating from the original concept.

Let the *system* referred to in the aforementioned classical reliability model be a CRN with one or multiple channels.



TFF = Time to first channel unavailability
$\mathrm{UT_i}$ = Channel available time
$\mathrm{DT_i}$ = Channel unavailable time
$\mathrm{CT_i}$ = $\mathrm{UT_i}$ + $\mathrm{DT_i}$ = Cycle period

Fig. 1: Illustration of channel availability time intervals and dependability metrics in CRNs.

The network is considered to be in the *operational mode* if there are enough channels available to commence a new SU request. A *system failure* represents the instant when there is no channel access opportunity for a newly arrived SU request to start its service. Correspondingly, the network is considered to be in the *failed mode* if the system has to block the new SU requests.

Similar to the concept of MUT, we define the mean channel available time, $\bar{T}_{UT}$, in a CRN as the average time duration during which the network resides in the operational mode. Accordingly, the mean channel unavailable time, $\bar{T}_{DT}$, is defined as the average time duration during which the network resides in the failed mode. Moreover, the MTFF in the considered CRN is represented by the mean time to first channel unavailability, $\bar{T}_{FF}$, indicating the expected time interval from network initiation to the instant when a new SU request is blocked for the first time. The concepts of UT, DT and TFF in CRNs as well as their relationships with channel availability are illustrated in Fig. 1. Note that the average values of $\mathrm{UT_i}$ and $\mathrm{DT_i}$ terms in Fig. 1, where $\mathrm{i} = 1, 2, 3, \cdots$, determine the $\bar{T}_{UT}$ and the $\bar{T}_{DT}$ values respectively.

C. Related Work

Although the notion of dependability has been an important aspect in communication systems since long time ago, very little work has been done on the dependability aspects of CRNs. In [10], a resource management controller was designed for primary and secondary traffic in vehicular networks. The proposed controllers in that paper provide reliable guarantees to PU traffic in terms of aggregate goodput and collision rate. Moreover, the same authors proposed a reliable adaptive resource management controller in [11] for cognitive cloud vehicular networks in order to provide hard reliability guarantees to PU traffic in the presence of mobility and fading induced changes in vehicular networks. However, the considered reliability metrics in those two papers are distinct from the metrics defined in this paper. To maximize the network performance of a *single channel CRN*, a combined scheme considering PU activity and spectrum hole detection was proposed in [12] to address the potential drawbacks of the Poisson model.

The availability and reliability of wireless multi-hop networks, not of CRNs, were evaluated in [13] by considering stochastic link failures. By placing redundant nodes at appropriate locations in the existing network, the availability of wireless links is improved. The availability of a multi-cell CRN under time varying channels was studied in [14] and accordingly an efficient spectrum allocation mechanism was proposed. In the proposed network model therein, the average number of available channels is determined based on busy probabilities of channels. The authors of [15] demonstrated that deterministic channel failure models might result in a significant over-estimation or under-estimation of network reliability. Therefore, a reliability assessment was performed for wireless mesh networks in [15] by considering a probabilistic regional failure model which analyzed the geographical location of the network. Another recent paper [16] focused on investigating the security-reliability tradeoff of cognitive relay transmission in the presence of realistic spectrum sensing. Therein reliability was characterized in terms of outage probability.

In [17], the probability of the availability for a spectrum band in CRNs was evaluated by incorporating the results of an algorithm which can predict the arrival patterns of PUs. Thereafter, a similar but improved technique was proposed by the same authors in [18] to evaluate channel availability in CRNs. In our earlier work [19], three availability metrics were defined from the perspective of dependability theory. In [20], the authors analyzed the feasibility of opportunistic spectrum access in CRNs to provide a robust infrastructure for wireless networks. Moreover, the work done in [21] and [22] also analyzed the reliability aspects in CRNs.

However, the research work presented in this paper is distinct from the above related work with respect to mainly two aspects. First, by defining important system times from *the perspective of dependability theory*, we propose a systematic approach for analyzing channel availability in multi-channel CRNs. Second, the presented work thoroughly analyzes channel availability of CRNs via multiple metrics including system times, asymptotic channel availability and the distribution of interval availability instead of studying merely the steady state availability as in [19] and [20]. To the best of our knowledge, this is the first work which defines the reliability metrics combined with channel access in multi-channel CRNs and develops accordingly mathematical models to analyze them from the dependability theory's perspective.

III. Network Scenario and Access Schemes

In this section, we present the network scenario considered in this study and the representative channel access schemes used to obtain numerical results to be presented in Sec. VI.

A. Network Scenario and Assumptions

In this study we consider both homogeneous and heterogeneous channel environments for a CRN. In the homogeneous channel scenario, all channels have equal bandwidth and equal transmission data rate per channel. In contrast, in the heterogeneous channel scenario, we consider two types of channels which have different bandwidth and transmission rates. In the following sections, CRNs with homogeneous channels

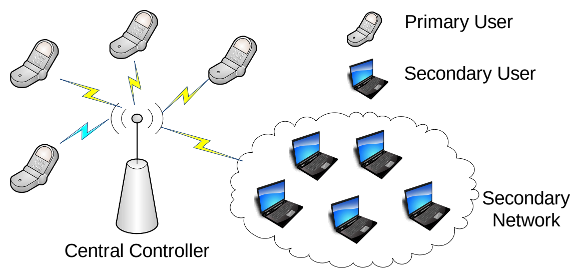


Fig. 2: A centralized architecture for CRN.

and heterogeneous channels are referred to as *Scenario I* and *Scenario II* respectively. In *Scenario I*, we consider a CRN with $M \in \mathbb{Z}^+$ channels with equal bandwidth where, $\mathbb{Z}^+$ is the set of positive integers. In *Scenario II*, a CRN with two types of channels is considered, namely, $M_1 \in \mathbb{Z}^+$ wider bandwidth channels hereafter referred to as wideband channels, and correspondingly $M_2 \in \mathbb{Z}^+$ narrowband channels. For presentation clarity, we denote the set of wideband channels as *WBC* and the set of narrowband channels as *NBC*.

In both scenarios, a centralized CRN architecture is considered. Fig. 2 illustrates the infrastructure based network architecture envisaged in this study. Resource allocation for the primary and secondary networks is coordinated by the base station which functions as the central controller in the network. Only one user can be allocated to a specific channel at a time. When a channel is occupied by an SU or a PU, the channel is said to be in the *busy mode* whereas it is in the *idle mode* if it is not occupied by any user. PUs have the priority of accessing all channels and SUs may occupy the channels which are in the idle mode in an opportunistic way. For user traffic considered in this study, we model traffic at the *flow level* since modeling at this level captures the dynamics related to the arrival and departure of flows such as flow duration or number of active flows. While a PU service occupies only one channel for each service, an SU may assemble several channels in order to complete its service at a higher data rate. Moreover, the following assumptions are made in order to develop our analytical model to be presented later.

- The arrivals of both PU and SU services are Poisson processes with arrival rates λ_P, λ_S for PU and SU services respectively.
- The service times for PU and SU services are exponentially distributed. In *Scenario I*, the service rates per channel for PU and SU services are μ_P and μ_S respectively. In *Scenario II*, the corresponding service rates per channel in the WBC are μ_{P1} and μ_{S1} respectively while those values in the NBC take μ_{P2} and μ_{S2} respectively.
- The sensing and spectrum adaptation latency is negligible in comparison with the time between two consecutive service events. These service events indicate PU and SU arrivals and departures.

It is worth mentioning that the proposed continuous time Markov chain (CTMC) models are applicable to other CRN channel access schemes as well, given that the PU and SU activity models are Markovian.

B. Dynamic Channel Access Scheme for Scenario I

1) Access scheme: The channel access scheme employed in the considered CRN with homogeneous channels is one of the dynamic channel access strategies proposed in [23], namely, dynamic fully adjustable (*DFA*). In the *DFA* scheme, channel aggregation is adopted, meaning that one SU service can utilize multiple channels if available. Two parameters, W and V, are introduced in *DFA* to represent the lower bound and the upper bound of the number of aggregated channels for an SU service respectively. Moreover, spectrum adaptation is allowed in order to protect ongoing SU services. Therefore, if an ongoing SU is interrupted *upon a PU arrival* and there are idle channels available, the interrupted SU will immediately release the occupied channel to the licensed PU and continue its service on one of the idle channels. In the following, we revisit the channel access procedure adopted in *DFA* with respect to the SU and PU activities.

- SU Arrival: When a new SU service arrives, the system should offer at least W channels to accommodate the new arrival. Otherwise the request is rejected. Moreover, the new SU service may aggregate up to V channels if there are enough channels available. If the number of idle channels upon an SU arrival is fewer than W, the ongoing SU service with the maximum number of channels will donate one or several channels as long as it can still keep W channels after donation. If the donated number is not enough, other ongoing SU services will collectively donate channels. If the number of idle channels plus the number of channels that can be donated by ongoing SU services is still fewer than W, the new SU request is blocked.
- PU Arrival: If a new PU arrives at a moment when there are idle channels in the CRN, the new PU can start transmission in an idle channel. If there is no idle channel, the SU service which has the maximum number of aggregated channels will donate one channel to the interrupted SU service, given that it has more aggregated channels than the interrupted SU service and its remaining number is still not fewer than W. In the worst case, if all ongoing SU services have exactly W channels upon a PU arrival, the interrupted SU service is forced to terminate.
- PU and SU Departures: As a result of *service departures of both PUs and SUs, or forced termination of SU services*, channels become idle. The ongoing SU service which has the minimum number of aggregated channels will then utilize those idle channels up to V. If the SU service with the minimum number reaches the upper bound V after spectrum adaptation and there are still idle channels, other SU services will occupy the remaining ones according to the same principle.

2) CTMC model for DFA: The states of the CTMC model corresponding to *DFA* can be represented by $\boldsymbol{x} = (i_{pu}, j_W, j_{W+1}, \cdots, j_V)$ where i_{pu} denotes the number of PU services in the CRN. The number of SU services with k aggregated channels in the CRN is denoted as j_k where $k = W, W+1, \cdots, V$. Let $\mathcal{S}_1$ be the set of feasible states

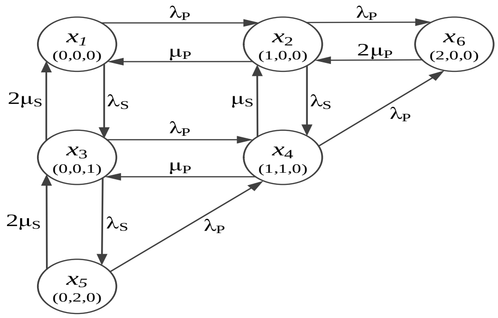


Fig. 3: State transition diagram of a CRN employing the *DFA* scheme when $M = 2, W = 1$ and $V = 2$ as well as the state transition rates.

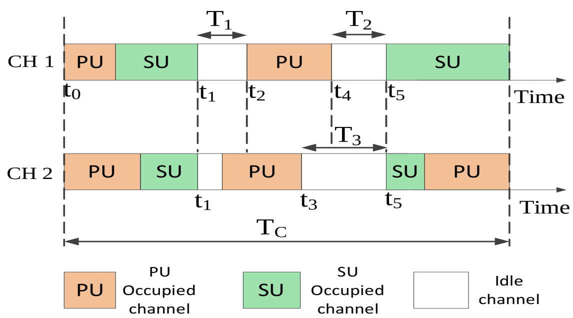


Fig. 4: Channel occupancy representation of a two channel CRN (*CH 1* and *CH 2*) where one SU service may occupy one or two channels if available.

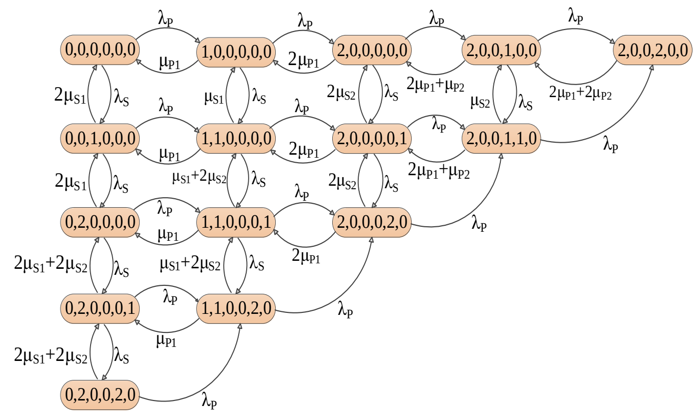


Fig. 5: State transition diagram for the *DFA$_H$* scheme when $M_1 = 2, M_2 = 2, W = 1, V = 2$ given that $\mu_{S1} > \mu_{S2}$.

of the system such that $\mathcal{S}_1 = \{\boldsymbol{x}|i_{pu}, j_W, j_{W+1}, \cdots, j_V \geq 0; b(\boldsymbol{x}) \leq M\}$ where $b(\boldsymbol{x}) = i_{pu} + \sum_{k=W}^{V} kj_k$. The state transitions associated with different service events are summarized by considering various traffic conditions in Table III.

C. An Example to Illustrate Channel Availability in DFA

To illustrate the system times and channel availability described in Subsec. II-A, we present an example topology of Scenario I. Consider a *DFA* employed CRN with $M = 2$ channels, where $W = 1$ and $V = 2$. Accordingly, to commence a new SU request, the CRN should be able to provide at least one channel. A general state, $\boldsymbol{x}$, of this system can be expressed as (i_{pu}, j_1, j_2), where i_{pu} denotes the number of PUs in the system, j_1 and j_2 denote the number of SUs which have single channel occupancy and two channel occupancy respectively. The states of this system are represented in Fig. 3 as $\boldsymbol{x_r}$ where $r = 1, 2, \cdots, 6$, and the state transitions corresponding to this example scenario are also shown.

Consider state $\boldsymbol{x_2}(1, 0, 0)$ where only one channel is occupied by a PU service. Indeed, it is a channel available state for new SU requests, since the other channel can be allocated upon a new SU request. Similarly, states $\boldsymbol{x_1}$ and $\boldsymbol{x_3}$ are also channel available states. However, in state $\boldsymbol{x_6}(2, 0, 0)$, both channels are occupied by two PUs and therefore $\boldsymbol{x_6}$ is *not* a channel available state for new SU requests. Likewise, states $\boldsymbol{x_4}$, and $\boldsymbol{x_5}$ are channel unavailable states since new SU services cannot be admitted to the network in those two states. An example of the channel occupancy status for this CRN during a continuous time interval T_C is shown in Fig. 4. As shown in this figure, a new SU service can be commenced with at least one channel from time instant t_1 to t_2 and time instant t_3 to t_5. Thus, during those time intervals the CRN is available for a new SU service. Let T_1 and T_3 represent the duration between time instants t_1 and t_2 and time instants t_3 and t_5 respectively, i.e., $T_1 = t_2 - t_1$ and $T_3 = t_5 - t_3$. In this paper, channel availability is regarded as *the fraction of time that the CRN can allocate at least the minimum number of required channels for a new SU request.* As defined in (3), the steady state channel availability of this CRN, A_{ss}, can be estimated as ${}^{(T_1 + T_3)}/_{T_C}$, given that the observation period T_C is sufficiently long.

D. Dynamic Channel Access Scheme for Scenario II

1) Access scheme: The basic principles of the channel access policy for *Scenario II* are the same as those highlighted in Sec. III-B for Scenario I, except the modifications explained herein. The proposed scheme for *Scenario II* is referred to as heterogeneous channel access for *DFA* (*DFA$_H$*).

Upon an arrival of a PU or an SU, the system first searches for an opportunity in the WBC. If there is no channel access opportunity in the WBC, then access in the NBC will be attempted. Although channel aggregation in *DFA$_H$* is also enabled, it is performed either among the WBC or the NBC channels. More complicated channel access which allows the aggregation of heterogeneous channels is not considered to avoid increased complexity in our model. Anyhow, we have also studied a more flexible channel access in CRNs by allowing both channel aggregation and channel fragmentation in another paper [24].

Upon a departure of a service or a forced termination of a service in the WBC, the resultant vacant channels are allocated according to the following rule. The first priority for accessing those vacant channels is given to the ongoing PUs and SUs in the NBC successively. Thus, the PUs and SUs in the NBC perform spectrum handover to the newly vacant channels in the WBC. However, the ongoing services in the NBC handover to the WBC only if the current aggregated service rate is lower than that of the new opportunity. If more vacant channels exist even after this handoff process, the SUs with minimum number of aggregated channels in the WBC can aggregate them. Nevertheless, the vacant channels in the NBC will *not* be allocated to SUs in the WBC considering that users with high bandwidth resources prefer to continue their services in the WBC rather than performing spectrum handover to a NBC and channel aggregation.

2) CTMC model for DFA$_H$: The states of the CTMC model can be represented by $\boldsymbol{x} = (i^w, j^w_W, j^w_{W+1}, \cdots, j^w_V, i^n, j^n_W, j^n_{W+1}, \cdots, j^n_V)$. Here i^w and j^w_k denote, respectively, the number of PU services and the number of SU services with $k(k \in \mathbb{Z}^+, W \leq k \leq V)$ aggregated channels in the WBC. Similarly, i^n and j^n_k denote the number of those services in the NBC. Let $\mathcal{S}_2$ be the set of feasible states of the system, $\mathcal{S}_2 = \{\boldsymbol{x} | i^w, j^w_W, j^w_{W+1}, \cdots, j^w_V, i^n, j^n_W, j^n_{W+1}, \cdots, j^n_V \geq 0; b_w(\boldsymbol{x}) \leq M_1; b_n(\boldsymbol{x}) \leq M_2\}$ where $b_w(\boldsymbol{x}) = i^w + \sum_{k=W}^{V} k j^w_k$ and $b_n(\boldsymbol{x}) = i^n + \sum_{k=W}^{V} k j^n_k$. The state transitions associated with different events in *DFA$_H$* are summarized with different conditions in Table IV. As an example, the complete state transition diagram for a CRN with $M_1 = 2, M_2 = 2, W = 1$ and $V = 2$, can be found in Fig. 5, which illustrates the state transitions associated with different PU and SU activities. Moreover, in Scenario II, channel availability is regarded as *the fraction of time that the CRN can allocate at least the minimum number of required channels for a new SU request either in the WBC or in the NBC.*

IV. CTMC Model for Dependability Analysis

CTMC models are quite often used as an analytical tool in reliability engineering [25]. Since it can capture the important interdependence and dynamic relationships between system components, it is more efficient than the other analytical techniques such as reliability block diagrams and fault tree analysis [26]. In this section, we present a stepwise procedure of the proposed analytical model to evaluate system times and channel availability based on CTMC analysis. Note that the analytical model developed in this section applies to both Scenario I and Scenario II.

A. State Space and Partitioned State Transition Matrix

The set of feasible states of the system is denoted as $\mathcal{S}$. Without loss of generality, assume that the total number of states in the system is N. Let i represent a general state in the system where $1 \leq i \leq N, \quad i \in \mathbb{Z}^+$. Here, $N = n(\mathcal{S})$, where $n(\mathcal{A})$ denotes the cardinality of the set $\mathcal{A}$. Furthermore, the steady state probability of being in state i is denoted as π_i. The stationary probabilities, π_i, can be calculated from the global balance equations and the normalization equation [27], which are given as

$$\boldsymbol{\pi Q} = \boldsymbol{0}, \quad \sum_{i \in \mathcal{S}} \pi_i = 1, \tag{4}$$

where $\boldsymbol{\pi}$ is the steady state probability vector, $\boldsymbol{Q}$ denotes the transition rate matrix, and $\boldsymbol{0}$ denotes a row vector of 0's of an appropriate size. Furthermore, the entire state space, $\mathcal{S}$, can be divided into two subsets, i.e., channel available states *(operational mode)* and channel unavailable states *(failed mode)*. For convenience, we consider that the states in $\mathcal{S}$ are ordered (lexicographically) and refer to them by their order number. In other words, the feasible states are re-arranged in a way that in subset $\mathcal{S}_\mathcal{A} = \{1, 2, ..., L\}$, the system can provide at least the minimum number of required channels for a new SU request, while in subset $\mathcal{S}_\mathcal{B} = \{L+1, ..., N\}$, the system failed to do so. Thus, $\mathcal{S} = \mathcal{S}_\mathcal{A} \cup \mathcal{S}_\mathcal{B}$ and $\mathcal{S}_\mathcal{A} \cap \mathcal{S}_\mathcal{B} = \varnothing$. Since $n(\mathcal{S}_\mathcal{A}) = L$, then $n(\mathcal{S}_\mathcal{B}) = N - L$.

By re-arranging the rows and columns, the transition rate matrix $\boldsymbol{Q}$ can be written in the following partitioned form

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix}, \tag{5}$$

where the $L \times L$ matrix $\boldsymbol{A}$ represents the transition rates from a state in $\mathcal{S}_\mathcal{A}$ to another state in $\mathcal{S}_\mathcal{A}$ and the $L \times (N-L)$ matrix $\boldsymbol{B}$ represents the transition rates from a state in $\mathcal{S}_\mathcal{A}$ to a state in $\mathcal{S}_\mathcal{B}$. Similarly, the $(N-L) \times L$ matrix $\boldsymbol{C}$ represents the transition rates from a state in $\mathcal{S}_\mathcal{B}$ to another state in $\mathcal{S}_\mathcal{A}$ and the $(N-L) \times (N-L)$ matrix $\boldsymbol{D}$ represents the transition rates from a state in $\mathcal{S}_\mathcal{B}$ to another state in $\mathcal{S}_\mathcal{B}$. Note that the entries in the main diagonal of $\boldsymbol{A}$ (and $\boldsymbol{D}$) represent the total outgoing rates from the states in $\mathcal{S}_\mathcal{A}$ (respectively, $\mathcal{S}_\mathcal{B}$) and, as such, they also include the transition rates to the states in $\mathcal{S}_\mathcal{B}$ (respectively, $\mathcal{S}_\mathcal{A}$). Let $P_i(t)$ be the probability that the system is in state i at time t. Define $\boldsymbol{P}_A(t) \equiv [P_1(t) \ P_2(t) \ \cdots \ P_L(t)]$ as an L-dimensional row vector with the transient probabilities of the channel available states with i^{th} component. Similarly, $\boldsymbol{P}_B(t)$ denotes the $N-L$ dimensional row vector representing the channel unavailable states, and $\boldsymbol{P}_B(t) \equiv [P_{L+1}(t) \ P_{L+2}(t) \ \cdots \ P_N(t)]$. Assume further that, at the system initial instant, i.e., $t = 0$, the system can provide sufficient number of idle channels to new SU requests, i.e.,

$$\boldsymbol{P}_A(0)\boldsymbol{U}_L = 1 \text{ and } \boldsymbol{P}_B(0)\boldsymbol{U}_{N-L} = 0, \tag{6}$$

where $\boldsymbol{U}_L$ denotes the column vector of L ones.

B. Distribution of the Time until the First Visit to Subset $\mathcal{S}_\mathcal{B}$

Let the system be initially at the state space, $\mathcal{S}_\mathcal{A}$. Once the system transits to one of the states in $\mathcal{S}_\mathcal{B}$ for the first time, the elapsed time from the system initiation until such a transition is the time to first channel unavailability. Therefore, in order to determine the distribution of the time until the first channel unavailability, we consider that all the states in $\mathcal{S}_\mathcal{B}$ as a *single absorbing state*. Now, consider a new CTMC in which all the states in $\mathcal{S}_\mathcal{B}$ are lumped into a single absorbing state $L+1$. Let the stochastic process $\{X(t) \in \mathcal{S} | t \in \mathbb{R}\}$ be a homogeneous Markov process defined on a discrete and finite state space $\mathcal{S} = \{1, 2, \cdots, L, L+1\}$. Therefore, in the new CTMC, $\mathcal{S}_\mathcal{A} = \{1, 2, \cdots, L\}$ and $\mathcal{S}_\mathcal{B} = \{L+1\}$. Since state $L+1$ is absorbing and all other states are transient, the generator matrix of the Markov chain can be written as,

$$\boldsymbol{Q}' = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{A}^0 \\ \boldsymbol{0} & 0 \end{bmatrix}. \tag{7}$$

In (7), $\boldsymbol{A}^0$ is a column vector such that $\boldsymbol{A}\boldsymbol{U}_L + \boldsymbol{A}^0 = \boldsymbol{0}_L$ where $\boldsymbol{0}_L$ denotes the column vector of L zeros. Let T be a random variable representing the time until the CTMC reaches the absorbing state. It is said that T follows a continuous phase-type (PH) distribution with the representation $(\boldsymbol{P}_A(0), \boldsymbol{A})$ [28]. For PH random variables, the probability distribution of the time until absorption in the Markov chain is given by $F_T(t) = 1 - \boldsymbol{P}_A(0)e^{\boldsymbol{A}t}\boldsymbol{U}_L$, and correspondingly the probability density function is expressed as,

$$f_T(t) = \frac{d}{dt}F_T(t) = \boldsymbol{P}_A(0)e^{\boldsymbol{A}t}\boldsymbol{A}^0. \tag{8}$$

The r^{th} moment about the origin of the time until absorption (time to system failure), m'_r, is given by

$$m'_r = E[T^r] = r!\boldsymbol{P}_A(0)(-\boldsymbol{A})^{-r}\boldsymbol{U}_L,$$

where $E[\mathcal{X}]$ denotes the *expected value* or *mean* of the continuous-type random variable $\mathcal{X}$. When $r = 1$, the mean value, i.e., m'_1, can be obtained as follows.

$$m'_1 = E[T] = \boldsymbol{P}_A(0)(-\boldsymbol{A})^{-1}\boldsymbol{U}_L. \tag{9}$$

To determine the mean time until the CTMC reaches the absorbing state, (9) can be used and it gives the basis for determining the system times introduced in the next subsection. Alternatively, Laplace transforms can also be used to derive (9), as presented in our earlier work [19]. In this paper, however, we adopt a simpler method to obtain (9) by means of lumping all absorbing states into a single state.

C. System Times and Steady State Channel Availability

In this subsection we derive analytical expressions for the dependability metrics introduced in Sec. II-B. In what follows, we give the mathematical definitions for the channel available and unavailable time in a CRN. Based on these time duration definitions, an expression for steady state channel availability is derived.

1) Mean time to first channel unavailability, $\bar{T}_{FF}$: It is defined as *the expected time from system initiation to the instant when a new user request is blocked by the system for the first time*. Denote now by $\boldsymbol{t} \equiv [t_1 \ t_2 \ \cdots \ t_L]$ the row vector representing the mean time until the CTMC reaches the absorbing state. Here t_i represents the mean total time spent in state i until absorption. The i-th row and j-th column element of $-\boldsymbol{A}^{-1}$ is the expected total time spent in state j during the time until absorption, given that the initial state is i. Then, by unconditioning the initial probabilities, we obtain

$$\boldsymbol{t} = \boldsymbol{P}_A(0)(-\boldsymbol{A})^{-1}, \tag{10}$$

given that the initial probability row vector of $\boldsymbol{P}_A(0) \equiv [P_1(0) \ P_2(0) \ \cdots \ P_L(0)]$. By solving (10), all the elements in $\boldsymbol{t}$ can be determined. Now, by summing up the elements in $\boldsymbol{t}$ (i.e., right multiplying by $\boldsymbol{U}_L$) we obtain

$$\bar{T}_{FF} = \boldsymbol{P}_A(0)(-\boldsymbol{A})^{-1}\boldsymbol{U}_L. \tag{11}$$

2) Mean time to channel unavailability, $\bar{T}_{TF}$: The mean time to channel unavailability is defined as *the average time interval from an instant when the channels are available for new SUs, to the next channel unavailable instant*. The channel available instant is assumed to be selected when the system is in the steady state. To calculate $\bar{T}_{TF}$, the steady-state probabilities of the state space are required. The steady state probability vector of $\mathcal{S}$, can be partitioned as $\boldsymbol{\pi} = [\boldsymbol{\pi}_A \ \boldsymbol{\pi}_B]$ where $\boldsymbol{\pi}_A \equiv [\pi_1 \ \pi_2 \ \cdots \pi_L]$ and $\boldsymbol{\pi}_B \equiv [\pi_{L+1} \ \pi_{L+2} \ \cdots \pi_N]$. In this subsection, our focus is to calculate the mean time to channel unavailability elapsed from an instant which is randomly chosen when the system is in a channel available state. Given that the system is in $\mathcal{S}_\mathcal{A}$, the probability that it is in state i is $(\boldsymbol{\pi}_A\boldsymbol{U}_L)^{-1}\pi_i$ and correspondingly, we can show that, $\boldsymbol{P}_A(0) = (\boldsymbol{\pi}_A\boldsymbol{U}_L)^{-1}\boldsymbol{\pi}_A$. Then following (9), we obtain

$$\bar{T}_{TF} = \frac{\boldsymbol{\pi}_A(-\boldsymbol{A})^{-1}\boldsymbol{U}_L}{\boldsymbol{\pi}_A\boldsymbol{U}_L}. \tag{12}$$

3) Mean channel available time, $\bar{T}_{UT}$: Once the system transits to a state in $\mathcal{S}_\mathcal{A}$ from a state in $\mathcal{S}_\mathcal{B}$, the system becomes available. *The average duration that the system resides in $\mathcal{S}_\mathcal{A}$ before making a transition back to a state in $\mathcal{S}_\mathcal{B}$* is defined as the mean channel available time. A channel available time duration will start in state $j \in \mathcal{S}_\mathcal{A}$, if the system was in any state $i \in \mathcal{S}_\mathcal{B}$ and then made a transition from state i to state j. Correspondingly, the probability of initiating a channel available time in state j can be expressed as,

$$P_j = \frac{\sum_{i=L+1}^{N}\pi_i q_{ij}}{\sum_{h=1}^{L}\{\sum_{i=L+1}^{N}\pi_i q_{ih}\}}, \tag{13}$$

where q_{ij} is the instantaneous transition rate from state i to state j. This result implies that

$$\boldsymbol{P}_A(0) = \frac{\boldsymbol{\pi}_B\boldsymbol{C}}{\boldsymbol{\pi}_B\boldsymbol{C}\boldsymbol{U}_L}. \tag{14}$$

Since $\boldsymbol{\pi}\boldsymbol{Q} = \boldsymbol{0}$, we have $\boldsymbol{\pi}_A\boldsymbol{A} + \boldsymbol{\pi}_B\boldsymbol{C} = \boldsymbol{0}$. Furthermore, as the sum of the elements in every row in $\boldsymbol{Q}$ is zero, we have $\boldsymbol{A}\boldsymbol{U}_L + \boldsymbol{B}\boldsymbol{U}_{N-L} = \boldsymbol{0}_L$. Therefore, we obtain

$$\boldsymbol{P}_A(0) = \frac{\boldsymbol{\pi}_A\boldsymbol{A}}{\boldsymbol{\pi}_A\boldsymbol{A}\boldsymbol{U}_L} = \frac{-\boldsymbol{\pi}_A\boldsymbol{A}}{\boldsymbol{\pi}_A\boldsymbol{B}\boldsymbol{U}_{N-L}}.$$

By substituting the above expression of $\boldsymbol{P}_A(0)$ into (9), we obtain

$$\bar{T}_{UT} = \frac{-\boldsymbol{\pi}_A\boldsymbol{A}(-\boldsymbol{A})^{-1}\boldsymbol{U}_L}{\boldsymbol{\pi}_A\boldsymbol{B}\boldsymbol{U}_{N-L}} = \frac{\boldsymbol{\pi}_A\boldsymbol{U}_L}{\boldsymbol{\pi}_A\boldsymbol{B}\boldsymbol{U}_{N-L}}. \tag{15}$$

4) Mean channel unavailable time, $\bar{T}_{DT}$: Once the system transits to a state in state space $\mathcal{S}_\mathcal{B}$ from a state in state space $\mathcal{S}_\mathcal{A}$, the system becomes unavailable. *The average time duration during which the system resides in $\mathcal{S}_\mathcal{B}$ before making a transition back to a state in $\mathcal{S}_\mathcal{A}$* is defined as the mean channel unavailable time. By following a similar procedure as in the previous $\bar{T}_{UT}$ calculation, we can derive that, $\bar{T}_{DT} = \frac{\boldsymbol{\pi}_B\boldsymbol{U}_{N-L}}{\boldsymbol{\pi}_B\boldsymbol{C}\boldsymbol{U}_L}$. Since we can show that, $\boldsymbol{\pi}_B\boldsymbol{C}\boldsymbol{U}_L = \boldsymbol{\pi}_A\boldsymbol{B}\boldsymbol{U}_{N-L}$, $\bar{T}_{DT}$ can be expressed as

$$\bar{T}_{DT} = \frac{\boldsymbol{\pi}_B\boldsymbol{U}_{N-L}}{\boldsymbol{\pi}_A\boldsymbol{B}\boldsymbol{U}_{N-L}}. \tag{16}$$

5) Steady state channel availability, A_{ss}: As already stated in (3), *the steady state availability is equal to the mean uptime divided by the summation of the mean uptime and the mean downtime*. Since $\bar{T}_{UT}$ and $\bar{T}_{DT}$ are analogous to the mean uptime and the mean downtime respectively, we show straightforwardly that

$$A_{ss} = \frac{\bar{T}_{UT}}{\bar{T}_{UT} + \bar{T}_{DT}} = \frac{\boldsymbol{\pi}_A\boldsymbol{U}_L}{\boldsymbol{\pi}_A\boldsymbol{U}_L + \boldsymbol{\pi}_B\boldsymbol{U}_{N-L}}. \tag{17}$$

Following the normalization equation in (4), $\boldsymbol{\pi}_A\boldsymbol{U}_L +$

$\boldsymbol{\pi}_B \boldsymbol{U}_{N-L} = 1$. Therefore, (17) is simplified as

$$A_{ss} = \boldsymbol{\pi}_A \boldsymbol{U}_L. \tag{18}$$

It is obvious that in order to calculate the above performance measures, we need to obtain the sets of channel available and unavailable states, the transition rate matrix and and their stationary probabilities, $\boldsymbol{\pi}$, of the system. It is worth mentioning that the procedure to obtain channel available and unavailable states and $\boldsymbol{\pi}$, does not rely on the adopted channel access scheme, as long as the PU/SU activity model is Markovian. An example which shows how to determine $\bar{T}_{DT}$ in a single channel CRN is illustrated in the Appendix.

V. Channel available time distributions

In the previous section we analyzed the steady state channel availability and several system times corresponding to the commonly used metrics in dependability studies. Moreover, channel availability can also be considered as an appropriate measure for QoS analysis in CR systems since the profit which the SN can gain depends on the channel availability level. However, the steady state availability alone is not sufficient when there is an associated penalty if *a pre-specified level of availability over a finite time interval is not met* [29]. In such a situation, the probability that the CRN cannot meet a specified channel availability level needs to be calculated. This observation triggered our motivation for further determining the distribution function of channel availability.

Note that the *interval availability* over a period $(0, t)$ is defined as *the fraction of time in which the system is in operation during* $(0, t)$, and it is another useful metric related to availability [26]. Hereafter, we present the details of a numerical approach which can determine the complementary cumulative distribution function of the channel availability in a CRN by computing *the probability of gaining an interval availability greater than a certain value, p*. Before presenting that approach, we briefly introduce a solution technique known as *uniformization* which can be used to numerically calculate the distribution of the channel availability during a finite observation period.

A. Uniformization Technique

Studying the transient regime of a CTMC involves solving a system of linear first-order differential equation, or equivalently evaluating a matrix exponential. Direct evaluation of a matrix exponential is not considered as a practical method to solve dependability models at the transient phase due to its high computation cost [30]. Alternatively, the *uniformization* or *randomization* method has been introduced to perform transient analysis of finite state CTMCs in many studies [31], [32]. The uniformization process can be applied to any discrete state Markov process with a bounded transition rate matrix $\boldsymbol{Q}$ and it reduces a CTMC to an equivalent discrete-time Markov chain (DTMC) subordinate to a Poisson process. In this method, the transition probability matrix $\boldsymbol{P}$ of the uniformized Markov chain is given by $\boldsymbol{P} = I + \boldsymbol{Q}/\Delta$, where Δ is referred to as the *uniformization rate* which is equal to or greater than the largest magnitude of the diagonal elements of $\boldsymbol{Q}$, i.e., $\Delta \geq \max\{|q_{ii}|, i \in \mathcal{S}\}$ and I denotes the identity matrix of the same size as $\boldsymbol{Q}$. For more details about uniformization, read [27] and [33]. In the next subsection we apply the uniformized Markov chain to calculate the interval availability distribution.

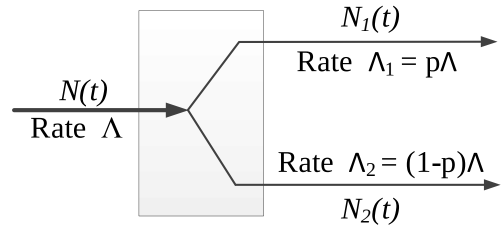


Fig. 6: Subdividing a Poisson process into two processes where each arrival is independently sent to process $N_1(t)$ with probability p and to process $N_2(t)$ with probability $1-p$.

B. Interval Availability Distribution

The states of a system at time t can be described by a CTMC, $X = \{X_t, t \geq 0\}$, over a discrete state space $\mathcal{S}$ [30]. The state space is assumed to be finite and of size N. As already mentioned in Sec. IV-A, the total state space can be divided into two disjoint subsets, $\mathcal{S_A}$ and $\mathcal{S_B}$. Let $O(t)$ be the cumulative amount of channel available time during $(0, t)$. We can express $O(t)$ as

$$O(t) = \int_0^t r(s)\, ds, \quad \text{where} \quad r(s) \triangleq \begin{cases} 1, & \text{if} \quad X_s \in \mathcal{S_A} \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

Then the interval availability of channel available time over $(0, t)$, $A_{int}(t)$, can be expressed as $A_{int}(t) = \frac{O(t)}{t}$. Consider that n transitions occurred during $(0, t)$ and therein the uniformized Markov chain visited $\mathcal{S_A}$, k times, where $0 \leq k \leq n+1$. Note that k can be equal to $n+1$ since initially the system may reside in one of the channel available states before any transition occurs. Let us now briefly recall how the cumulative distribution function of the interval availability is derived [33], [35].

Consider a Poisson counting process, $\{N(t), t \geq 0\}$, split into two processes $\{N_1(t); t \geq 0\}$ and $\{N_2(t), t \geq 0\}$: each arrival in $\{N(t), t \geq 0\}$ is sent to the first process with probability p and to the second process with probability $1-p$ (see Fig. 6). Out of the first n arrivals to the combined process, the probability that k or more arrivals occur to the first process is given by $\sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}$ [34]. Consider that the above mentioned first and second processes are the uniformized Markov chain visits to $\mathcal{S_A}$ and $\mathcal{S_B}$ respectively. By following these properties of the Poisson process, it can be shown that the following conditional probability holds

$$P\left(A_{int}(t) \leq p \,\middle|\, \begin{array}{c} n \text{ state changes in } (0,t) \\ k \text{ or more visiting occurrences to } \mathcal{S_A} \end{array}\right) = \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}, \tag{20}$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, and $p < 1$ is a pre-specified value of the required channel availability for the system. Hereafter, we refer to the parameter, p, as the *guaranteed level* of channel availability. To remove the condition on the number of visiting occurrences to the states of $\mathcal{S_A}$, define $Y_{n,k}$, $0 \leq k \leq n+1$ to be the probability that the uniformized Markov chain visits $\mathcal{S_A}$, k times before the $(n+1)^{th}$ transition occurs. Unconditioning on the number of visits to $\mathcal{S_A}$, we have

$$P(A_{int}(t) \leq p|\ n \text{ state changes in } (0,t)\ ) = \sum_{k=0}^{n} Y_{n,k} \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}. \quad (21)$$

When uniformization is applied to the CTMC, we have a Poisson process with rate equal to the uniformization rate Δ. Considering this Poisson process, we can further perform unconditioning on the number of transitions in $(0,t)$ since parameter n is a counting process in $(0,t)$. Then we obtain,

$$P(A_{int}(t) \leq p) = \sum_{n=0}^{+\infty} e^{-\Delta t} \frac{(\Delta t)^n}{n!} \sum_{k=0}^{n} Y_{n,k} \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}. \quad (22)$$

It is obvious that the infinite sum in (22) cannot be implemented for the numerical evaluation. However an approximate solution with adequate accuracy is sufficient for CCDF evaluation. Therefore, we truncate the above series by limiting the outer summation to $N_c + 1$ terms such that,

$$P(A_{int}(t) \leq p) = err(N_c) + \sum_{n=0}^{N_c} e^{-\Delta t} \frac{(\Delta t)^n}{n!} \sum_{k=0}^{n} Y_{n,k} \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}, \quad (23)$$

where, $err(N_c)$ is estimated as the first term left out of the summation, i.e., the term $n = N_c + 1$, and that N_c has been chosen so that $err(N_c)$ is sufficiently small (with $err(N_c) \leq 10^{-4}$) and it is given as

$$err(N_c) = \sum_{n=N_c+1}^{+\infty} e^{-\Delta t} \frac{(\Delta t)^n}{n!} \sum_{k=0}^{n} Y_{n,k} \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}. \quad (24)$$

Consequently, we can deduce that

$$P(A_{int}(t) \leq p) \approx \sum_{n=0}^{N_c} e^{-\Delta t} \frac{(\Delta t)^n}{n!} \sum_{k=0}^{n} Y_{n,k} \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}. \quad (25)$$

In this study, N_c is selected in an iterative way by calculating $P(A_{int}(t) \leq p)$ corresponding to certain values of N_c and $N_c + 1$. Once we obtain $P_{N_c+1} - P_{N_c} \leq 10^{-4}$, the calculation process is terminated and the corresponding N_c is selected. Here P_{N_c} denotes the value of $P(A_{int}(t) \leq p)$ after the series is truncated into N_c terms.

C. Calculation of $Y_{n,k}$ and CCDF of Channel Availability

Based on (25), the only parameter left to calculate $P(A_{int}(t) \leq p)$ is $Y_{n,k}$. In this study, we revise the approach proposed in [35] to recursively determine $Y_{n,k}$. Firstly, we define $\boldsymbol{Y}^{\mathcal{S}}_{n,k}$ as a row vector of N entries where N is the size of the total state space $\mathcal{S}$. The i^{th} entry of $\boldsymbol{Y}^{\mathcal{S}}_{n,k}$ is the probability that the uniformized Markov chain visits the states of $\mathcal{S_A}$, k times during its first n transitions and the n^{th} transition occurs into state i. We partition $\boldsymbol{Y}^{\mathcal{S}}_{n,k}$ according to the subsets $\mathcal{S_A}$ and $\mathcal{S_B}$ as $\boldsymbol{Y}^{\mathcal{S}}_{n,k} = [\boldsymbol{Y}^{\mathcal{S_A}}_{n,k},\ \boldsymbol{Y}^{\mathcal{S_B}}_{n,k}]$ where $\boldsymbol{Y}^{\mathcal{S_A}}_{n,k}$ is a row vector with L entries and $\boldsymbol{Y}^{\mathcal{S_B}}_{n,k}$ is a row vector with $N-L$ entries. Let $\boldsymbol{P}(0)$ represent the initial state probability vector such that $\boldsymbol{P}(0) = [\boldsymbol{P}_A(0),\ \boldsymbol{P}_B(0)]$ where $\boldsymbol{P}_A(0)$ and $\boldsymbol{P}_B(0)$ have the same meaning as mentioned in Sec. IV-A. Then the following initial conditions can be formulated,

$$\boldsymbol{Y}^{\mathcal{S_A}}_{0,1} = \boldsymbol{P}_A(0),\quad \boldsymbol{Y}^{\mathcal{S_B}}_{0,0} = \boldsymbol{P}_B(0),\quad \boldsymbol{Y}^{\mathcal{S_A}}_{0,0} = \boldsymbol{0},\quad \boldsymbol{Y}^{\mathcal{S_B}}_{0,1} = \boldsymbol{0}. \quad (26)$$

For instance, $\boldsymbol{Y}^{\mathcal{S_A}}_{0,1}$ in (26) is the probability that the uniformized Markov chain visits one channel available state without any state transition occurred (i.e., $k = 1$ and $n = 0$). This could happen only if the system resides in one of the channel available states initially. Thus, $\boldsymbol{Y}^{\mathcal{S_A}}_{0,1}$ equals to the initial probability vector, $\boldsymbol{P}_A(0)$. The following recursion formulas are used to calculate the whole set of $\boldsymbol{Y}^{\mathcal{S}}_{n,k}$.

$$\boldsymbol{Y}^{\mathcal{S_A}}_{n,k} = \boldsymbol{Y}^{\mathcal{S_A}}_{n-1,k-1}\boldsymbol{A'} + \boldsymbol{Y}^{\mathcal{S_B}}_{n-1,k-1}\boldsymbol{C'}, \quad (27)$$

$$\boldsymbol{Y}^{\mathcal{S_B}}_{n,k} = \boldsymbol{Y}^{\mathcal{S_A}}_{n-1,k}\boldsymbol{B'} + \boldsymbol{Y}^{\mathcal{S_B}}_{n-1,k}\boldsymbol{D'}, \quad (28)$$

where $\boldsymbol{A'}$, $\boldsymbol{B'}$, $\boldsymbol{C'}$ and $\boldsymbol{D'}$ are the sub-matrices of the uniformized Markov chain corresponding to (5) in Sec. IV -A.

In order to understand the reasoning behind the above mentioned recursive formulas, we analyze (27) as an example. The i-th element of $\boldsymbol{Y}^{\mathcal{S_A}}_{n,k}$ in (27) is the probability of the uniformized Markov chain being k times in channel available (operational) states out of n transitions in total, and the state visited after the last transition is the i-th state of $\mathcal{S_A}$. The last transition can occur from a state either within $\mathcal{S_A}$ or from $\mathcal{S_B}$. If it occurs from $\mathcal{S_A}$, the corresponding transition probabilities are given by $\boldsymbol{A'}$ and if it occurs from a state in $\mathcal{S_B}$, the corresponding transition rate is $\boldsymbol{C'}$. In both cases, the number of transitions increases by one due to the state transition. Correspondingly, the number of visited operational states increases by one since the transition is into $\mathcal{S_A}$. In contrast, in (28), the sub-index n is increased by one, and k is not increased. The reason is that the n^{th} transition occurred into a channel unavailable state in $\mathcal{S_B}$. Once $\boldsymbol{Y}^{\mathcal{S}}_{n,k}$ is determined, we obtain

$$Y_{n,k} = \boldsymbol{Y}^{\mathcal{S}}_{n,k}\boldsymbol{U}_N, \quad (29)$$

where $\boldsymbol{U}_N$ denotes the column vector of N ones. After the above calculation, $P(A_{int}(t) \leq p)$ in (25) can be deduced. Finally, the CCDF of the channel availability given by $A(t,p)$ is obtained by

$$A(t,p) = P(A_{int}(t) > p) = 1 - P(A_{int}(t) \leq p). \quad (30)$$

VI. Numerical Results and Discussions

In this section we evaluate numerically each dependability measure presented in Sec. IV and Sec. V based on the proposed analytical models. The numerical results obtained from the analytical models are validated through extensive simulations which are performed by using a custom-built discrete-event simulator based on MATLAB. The centralized CRN topology described in Sec. III is considered. The network has $M = 6$ channels and the default values of the parameters mentioned in Sec. III-A are configured as $\lambda_P = 1.0$, $\lambda_S =$

TABLE I: Four initial conditions used for performance evaluation of *DFA* (the set of feasible states of the system is denoted as $\mathcal{S} = (y, x_1, x_2, x_3)^\dagger$ where $M = 6$, $W = 1$ and $V = 3$.)

Initial condition	*DFA* states, (y, x_1, x_2, x_3)
C1: All channels are idle	$(0, 0, 0, 0)$
C2: One PU service and one SU service are in the system	$(1, 0, 0, 1)$
C3: System can allocate at most W channels upon a new SU arrival	$(y, x_1, 1, 0)^\ddagger$ and $(5, 0, 0, 0)$
C4: Only one PU service is in the system	$(1, 0, 0, 0)$

† Here y denotes the number of PU services each with single channel occupancy and x_k denotes the number of SU services with $k(1 \leq k \leq 3)$ aggregated channels.
‡ There are five states which satisfy $y + x_1 = 4$: $(0, 4, 1, 0)$, $(1, 3, 1, 0)$, $(2, 2, 1, 0)$, $(3, 1, 1, 0)$ and $(4, 0, 1, 0)$.

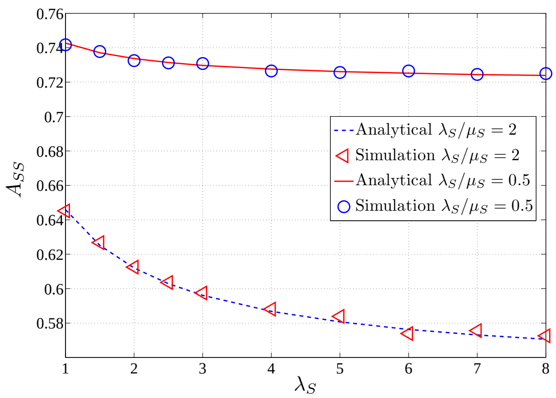


Fig. 7: A_{SS} as a function of λ_S while keeping the offered traffic, λ_S/μ_S, constant.

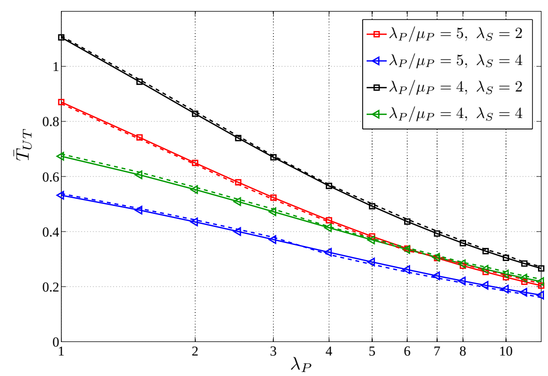


Fig. 8: $\bar{T}_{UT}$ as a function of λ_P while keeping the offered traffic, λ_P/μ_P, constant. The dashed lines indicate the simulation results.

2.0, $\mu_P = 0.5$ and $\mu_S = 1.0$ respectively. The units of these parameters could be services or flows per unit of time. *DFA* or *DFA$_H$* explained in Sec. III-B is employed as the channel access scheme, with $W = 1$ and $V = 3$. In order to analyze the system from different angles, various initial conditions of the system are configured. The initial system conditions adopted in this study are listed in Table I. Unless otherwise stated, *initial condition C1* is adopted in numerical evaluations. In Subsec. VI-K, *DFA$_H$* is employed in *Scenario II* whereas all other subsections in Sec. VI are dedicated to analyze *Scenario I* based on *DFA*.

A. Steady State Channel Availability

Fig. 7 shows the steady state channel availability variation as a function of SUs' arrival rate when the offered SU traffic load, i.e., the ratio $\frac{\lambda_S}{\mu_S}$, is constant. This figure illustrates A_{SS} for two different offered traffic values. The results in Fig. 7 show that higher channel availability is achieved in the CRN when the offered traffic of SUs is low. A low offered traffic load implies that the channels are less likely to be occupied by existing users. As a result, the new SU arrivals have more channel access opportunities. Therefore it is evident that A_{SS} holds a comparatively high value when $\frac{\lambda_S}{\mu_S}$ is low. However, the channel availability decreases with a higher SU arrival rate although the offered traffic load is kept constant. Given that the offered load, i.e., λ_S/μ_S, is constant, SUs would utilize the amount of resources (measured as mean number of occupied channels) if they had unconstrained access to the channels.

However, the number of channels is limited and SUs can only access the system resources when those are not occupied by PUs. As a consequence of keeping the offered load constant but with more frequent requests (a higher λ_S) and shorter service duration (a higher μ_s), the likelihood that SUs complete their services before they are forcibly terminated is increased. Thus a lower forced termination probability is achieved. This reduction of the forced termination probability clearly increases the carried load by the system, leading to lower A_{SS}. At higher offered traffic loads, the above mentioned observation is more evident as can be observed from Fig. 7.

To validate the correctness of the mathematical analysis, we present the simulation results obtained for channel availability measures in Figs. 7-9 and Fig. 12 together with the analytical results. It is worth mentioning that the simulations are performed independently of the developed analytical models. In other words, the calculation of the system times and availability values in our simulations is not dependent on the derived mathematical expressions at all, nor are the state transition tables used in these calculations. From these curves, we observe that the results from the analytical model coincide precisely with the obtained simulation results. Later on in Sec. VI-G, we further check the preciseness of the model by employing different PU interarrival time and SU service time distributions.

B. Mean Channel Available Time

Next we investigate the impact of the PU arrival rate on mean channel available time, $\bar{T}_{UT}$, which corresponds to MUT in classical reliability analysis. Fig. 8 shows the mean channel available time variation as a function of PUs' arrival rate when the PU offered traffic load, i.e., the ratio $\frac{\lambda_P}{\mu_P}$, is constant. As illustrated in this figure, the mean channel available time decreases as the offered traffic load becomes higher. The reasons for this behavior are explained below. When the

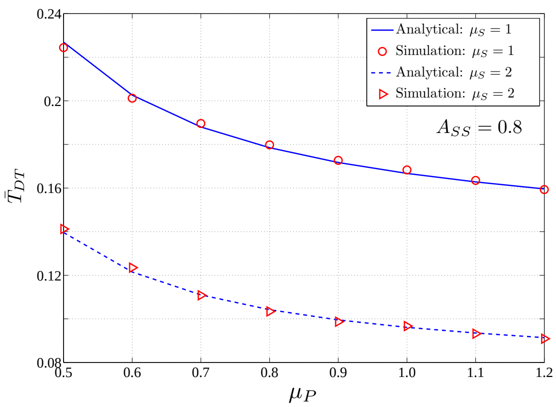


Fig. 9: $\bar{T}_{DT}$ as a function of μ_P when $A_{ss} = 0.8$.

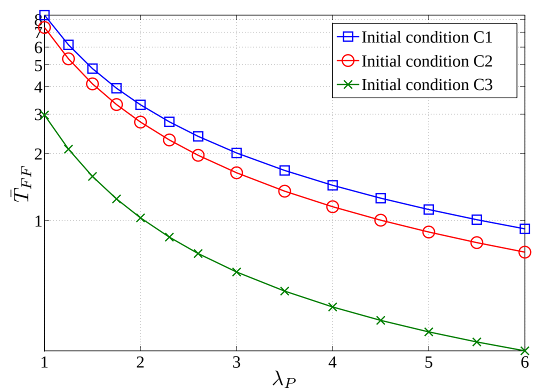


Fig. 10: $\bar{T}_{FF}$ as a function of λ_P. For the definitions of initial conditions C1, C2 and C3, refer to Table I.

offered traffic load is high, the system has a large number of active PU services. The time duration in which a particular channel being in the idle state, i.e., the available time of the channel, becomes shorter when PUs become more active. The results also clearly demonstrate that the channel available time duration becomes longer when λ_S is at a lower value due to the same reason. The reason for the descending trend of $\bar{T}_{UT}$ with a larger λ_P value can be explained in a similar way as described in the previous subsection.

C. Mean Channel Unavailable Time

The mean downtime of a system is a critical parameter since long downtime duration causes dissatisfaction of customers. The mean channel unavailable time as a function of the PU service rate is plotted in Fig. 9, given that the steady state channel availability of the system is set as 0.8 by adjusting the SU arrival rate, λ_S. From this figure, we observe a continuous descent in the channel unavailable time as the service rate of PUs increases while keeping the arrival rate of PUs constant. When the PU service rate is low, fewer PUs finish their services per unit of time. Then, most of the channels are occupied by respective PUs for a longer period of time. That means, the time interval that the system will reside in the channel unavailable state space is comparatively long. It is clear that when the μ_P is high, the probability of being in the space $\mathcal{S}_{\mathcal{B}}$ is low, thus the value of $\boldsymbol{\pi}_{\mathcal{B}} \boldsymbol{U}_{N-L}$ in (16) becomes low, resulting in a shorter $\bar{T}_{DT}$.

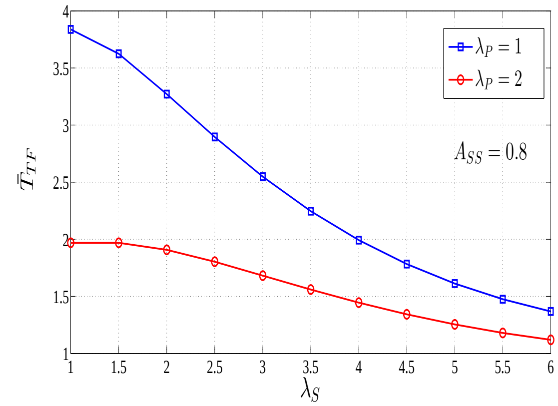


Fig. 11: $\bar{T}_{TF}$ as a function of λ_S when $A_{ss} = 0.8$.

On the other hand, at a higher PU service rate, a comparatively large number of PU services can be finished during a unit of time. Thus, the time period that the system will reside in the channel unavailable state space becomes shorter with a higher μ_P. For this reason, the mean channel unavailable time becomes shorter when PUs finish their services faster, as shown in Fig. 9. Additionally, for a given μ_P value, the channel unavailable time is longer at a lower SU service rate compared with a larger SU service rate. Again, this is due to the fact that the state holding time of the system being in a busy state decreases with a higher μ_S.

D. Mean Time to First Channel Unavailability

In Fig. 10, the plots indicate the mean time to first channel unavailability corresponding to the three initial conditions indicated in Table I. At *C1*, all channels are in the idle state while at *C2*, fewer channels are available in the beginning. At *C3*, the system can allocate merely $W = 1$ channel upon a new SU arrival. Thus, this initial condition, *C3*, represents a sort of *worst-case* scenario. As expected, when PUs arrive more often, all $\bar{T}_{FF}$ curves decrease monotonically. This is because that at lower PU arrival rates, more channels are likely to be idle and the newly arrived user requests can be accommodated with the required number of channels. Therefore, the network can operate without blocking any new users over a longer period. This result is in sharp contrast with the result under the high PU arrival rate circumstances.

As mentioned earlier, the time taken for the first channel unavailability is depending on the initial state of the system. If the system does not have any existing services in the beginning, $\bar{T}_{FF}$ will last longer compared with the system with already commenced PU and/or SU services, as illustrated in Fig. 10. When a single PU service and a single SU service already exist in a CRN with 6 channels, 4 channels are already occupied at the system initialization in *DFA* since $V = 3$. In other words, only 2 idle channels are available in the network for new users. As a consequence, there is a higher probability that this system will move to a channel unavailable state within a shorter period of time in comparison with the system with 6 idle channels in the initial phase.

E. Mean Time to Channel Unavailability

Fig. 11 presents the results for the mean time to channel unavailability, $\bar{T}_{TF}$, with respect to the arrival rate of SUs

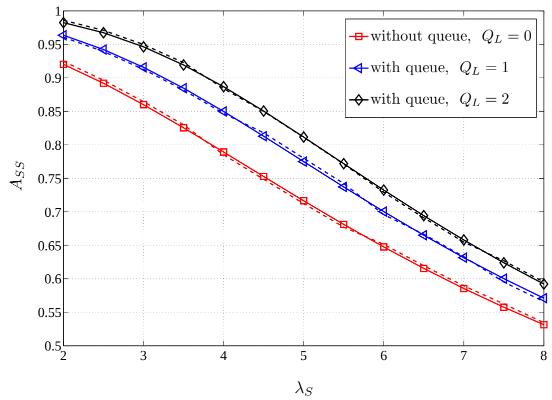


Fig. 12: A_{SS} as a function of λ_S for *DFA* and *DFAQ* schemes. The dashed lines indicate the simulation results.

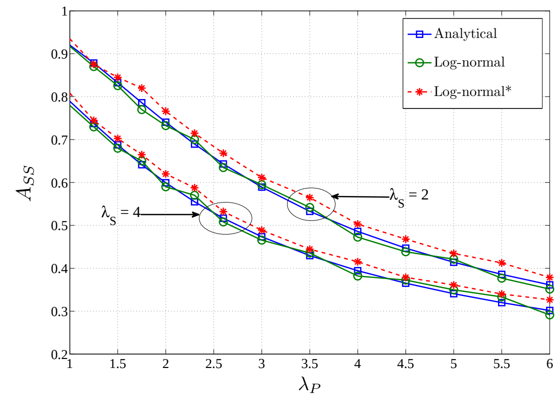


Fig. 13: Steady state availability, A_{SS}, as a function of the primary user arrival rate when the log-normal distribution is assumed for SU service time and PU interarrival time. The distributions of time sequences in the different experiments are listed in Table II.

when $A_{SS} = 0.8$. As expected, the mean time for the next system failure (channel unavailability) also shows a descending trend with a higher SU arrival rate. For a given steady state channel availability, the higher the number of SU arrivals to the system, the higher the channel occupancy. As a consequence of high channel occupancy, the remaining time in the channel available state space will reduce. In other words, at higher channel occupancy, the mean total time spent in $\mathcal{S}_\mathcal{A}$ and the steady state probabilities in $\boldsymbol{\pi}_A$ show lower values. Then the numerical value of the term $\boldsymbol{\pi}_A(-\boldsymbol{A})^{-1}\boldsymbol{U}_L$ in (12) is lower, leading to the decreasing trend of $\bar{T}_{TF}$ with a larger λ_S. In order to investigate the impact of PU arrival rate on the behavior of $\bar{T}_{TF}$, Fig. 11 depicts $\bar{T}_{TF}$ for two different PU arrival rates. As can be observed from this figure, $\bar{T}_{TF}$ further reduces when more PUs arrive.

F. Channel Availability Improvement via a Queuing Scheme

So far we investigate the channel availability as a reliability metric by considering the CRN as a *loss system*. In this subsection we adopt a queuing scheme which is proposed in our earlier work in [36] as a technique to improve channel availability of CRNs. We deploy a first in first out queue for the newly arrived secondary traffic flows mentioned in the *DFA* scheme. Note that this modified scheme is denoted as *DFAQ* in this paper for illustration convenience and the queue has a finite buffer size, Q_L. Using *DFA* discussed in Sec. III-B, a new SU request is blocked once the network is fully occupied.

However, with the proposed queuing scheme *DFAQ*, the new SU request which would be blocked by *DFA* is fed back into the buffer until the buffer is full. If the queue is fully occupied, then the new request is blocked. As we already mentioned, the derived analytical models in Sec. IV-C is independent of the channel access scheme since the derived mathematical expressions in that section do not depend on the parameters of the channel access scheme. Correspondingly, in order to analyze the steady state channel availability in *DFAQ*, we need only to determine the correspondent subsets, $\mathcal{S}_\mathcal{A}$ and $\mathcal{S}_\mathcal{B}$ of *DFAQ*, and evaluate the steady state probability vector $\boldsymbol{\pi}$ by using (4). Then, the summation of the steady state probabilities of $\mathcal{S}_\mathcal{A}$, i.e., $\sum_{i\in\mathcal{S}_\mathcal{A}}\pi_i$, equals to the steady state channel availability of *DFAQ* as given in (18).

Fig. 12 depicts the comparison between the *DFA* scheme and the *DFAQ* scheme on the performance of steady state channel availability as a function of the SU arrival rate. From this figure, we observe a monotonic descent for channel availability as the arrival rate of SUs increases. Furthermore, the channel availability using *DFA* is lower than that of *DFAQ* for the same configuration. Clearly, the channel availability becomes higher using the queuing scheme since the blocked services due to insufficient channels are buffered into the queue until they are possibly offered with the required number of channels. With a larger queue size, more SUs can be queued instead of being blocked, resulting in higher channel availability. However, this higher channel availability is achieved at a cost of queuing delay [36]. Therefore, the buffer size should not be too large in order to minimize the associated queuing delay.

TABLE II: The distribution of time sequences in the different experiments (cases) illustrated in Fig. 13.

Case	PU ST	SU ST	PU IAT	SU IAT
Analytical	EXP	EXP	EXP	EXP
Log-normal	LN	LN	EXP	EXP
Log-normal*	EXP	EXP	LN	EXP

ST and *IAT* in this table denote service time and interarrival time respectively. *EXP* and *LN* denote the exponential distribution and log-normal distribution respectively.

G. Steady State Channel Availability for Traffic Pattern with Log-normal Distribution

To further assess the preciseness as well as the applicability of the developed model which is based on the assumption of exponential service time and interarrival time (Poisson arrival process), we adopt another distribution and obtain the steady state availability by simulations. Log-normal distribution is a more realistic model for service times of real-life traffic patterns [37] as well for interarrival times. In Fig. 13, the steady state availability is depicted as a function of the *PU arrival rate based on the log-normal distribution* with the same mean value and the variance as used in the original exponential distribution for service times. For PU interarrival times, although the mean value of the log-normal distribution is kept the same as in the original exponential distribution,

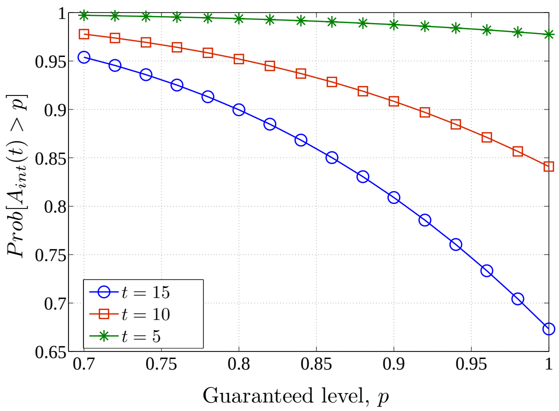


Fig. 14: Probability of achieving at least p channel availability as a function of the guaranteed level.

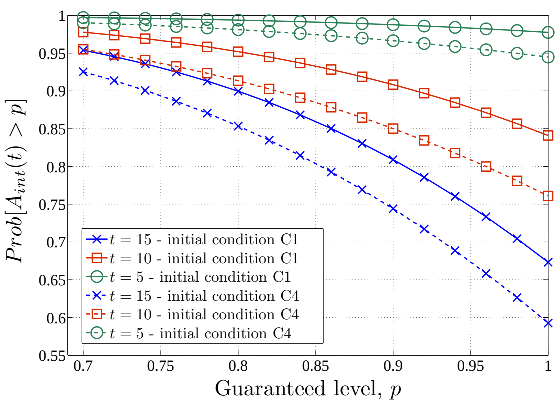


Fig. 15: Probability of achieving at least p channel availability as a function of the guaranteed level under different initial conditions. For the definitions of initial conditions C1, C2 and C3, refer to Table I.

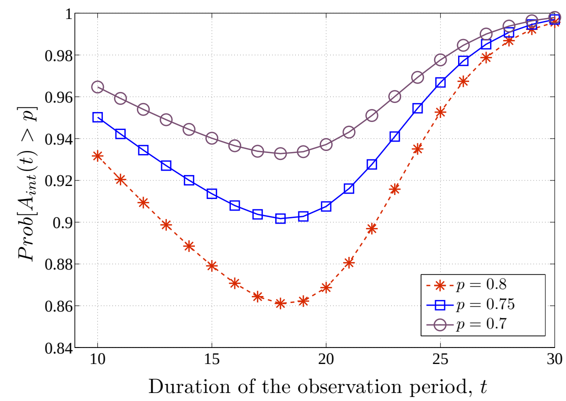


Fig. 16: Probability of achieving at least p channel availability when $p < A_{ss}$ as a function of observation period.

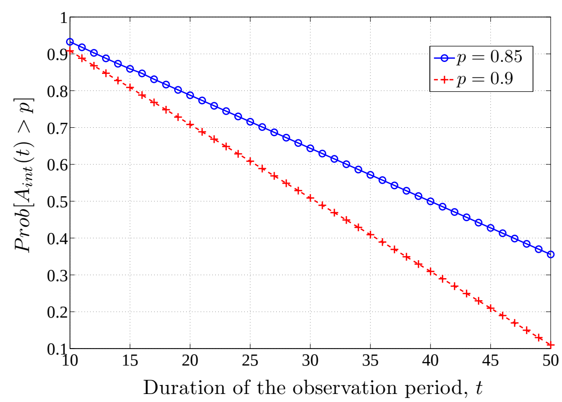


Fig. 17: Probability of achieving at least p channel availability when $p > A_{ss}$ as a function of observation period.

the variance is larger as its squared coefficient of variation ($SCV = {}^{\text{variance}}/_{\text{mean}^2}$) varies such that $1 \leq \text{SCV} \leq 4.6$. For comparison, we depict also the corresponding analytical results which are obtained based on the original exponential distribution in the same figure. The curves obtained from analyses and simulations match with each other precisely, as illustrated in Fig. 13, confirming that the proposed model is robust and applicable to other service time distributions and non-Poisson PU arrival processes as well.

In the above subsections from VI-A to VI-G, we have investigated the dependability metrics related to system times and the *steady state availability* of a CRN. In the next two subsections, the CCDF of the *interval availability* is studied as a function of the guaranteed level, p, and the duration of the observation period, t, respectively. *The purpose of those two subsections is to analyze the CRN's ability to satisfy the requirements of the SN on channel availability over a given period.* The numerical results presented in the following subsections are obtained by selecting the truncation parameter introduced in (23) such that $P_{N_c+1} - P_{N_c} \leq 0.0001$.

H. Channel Availability Distribution as a Function of a Guaranteed Level

In Fig. 14, the behavior of the CCDF for channel availability obtained by (30) is illustrated with different observation time periods where t represents the observation time in time units. As shown in the figure, the probability of spectrum availability for the SN for a guaranteed level monotonically decreases with a growing guaranteed level p. For instance, the probability that the spectrum will be available for more than 75% of the 15 time units during the observation period is around 0.93. However, this probability decreases to 0.81 if we need more than 90% of channel availability. In other words, the possibility of obtaining a higher channel availability level becomes lower when the system requires a higher availability level. Note that the probability of spectrum availability for higher than a specific guaranteed level also decreases when the duration of the observation period, i.e., t, becomes longer. The reason for this behavior can be explained as follows. If we look at an extreme case, i.e., an observation period of 0 time units, the entire spectrum is available for SUs due to the initial condition of idle channels. As t lasts longer, however, part of the spectrum will likely be occupied by PUs and/or SUs, leading to lower channel availability.

In order to further investigate the above observation, each curve presented in Fig. 14 is analyzed under two different initial conditions as shown in Fig. 15. The solid lines in Fig. 15 represent the CCDF at *C1* while the dashed lines illustrate the CCDF at *C4*. In contrast to *C1*, the channel availability for the SUs in *C4* is lower since one channel is already occupied in the beginning according to *C4*. Thus the probability of achieving

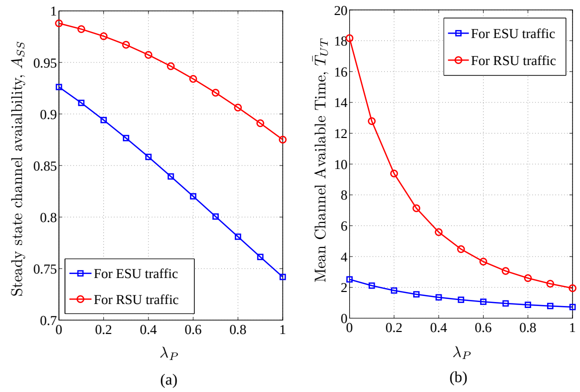
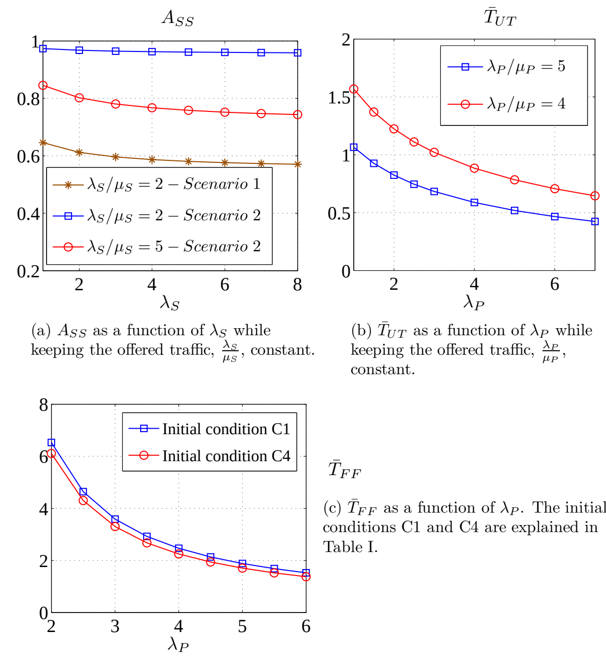


Fig. 18: Steady state availability and mean channel available time for hybrid SU traffic when $M = 6$.



(a) A_{SS} as a function of λ_S while keeping the offered traffic, $\frac{\lambda_S}{\mu_S}$, constant.

(b) $\bar{T}_{UT}$ as a function of λ_P while keeping the offered traffic, $\frac{\lambda_P}{\mu_P}$, constant.

(c) $\bar{T}_{FF}$ as a function of λ_P. The initial conditions C1 and C4 are explained in Table I.

Fig. 19: Channel availability analysis in the heterogeneous channel scenario. Please note that Scenario I in Fig. 19(a) represents the homogeneous channel scenario employed in Fig. 7.

channel availability higher than a specified guaranteed level under *C4* is always lower than that under *C1*.

I. Channel Availability Distribution as a Function of Observation Time

Furthermore, we show the probability of achieving a channel availability value which is greater than p, i.e., $P(A_{int}(t) > p)$, as a function of the duration of the observation period for different values of p. Basically two different types of behavior can be observed depending on whether the guaranteed level p can be reached in the asymptotic case ($p < A_{ss}$) or not ($p \geq A_{ss}$). Fig. 16 and Fig. 17 illustrate channel availability distribution as a function of the observation time under *C1*. In both illustrations *the steady state channel availability, A_{ss}, of the particular system is* fixed as 0.82.

1) When $p < A_{ss}$: Fig. 16 depicts $P(A_{int}(t) > p)$, as a function of the observation period when $p < A_{ss}$. As shown in those curves, the probability first decreases as the duration of the observation period becomes longer and then it increases up to one. Since the whole spectrum is available at time 0, $P(A_{int}(t) > p)$ is close to one for small values of t. As the duration of the observation period lasts longer, i.e., with a larger value of t, the possibility of channel access decreases due to the commencement of PU and SU services. That is, both channel availability and $P(A_{int}(t) > p)$ decrease with a larger t. However, further extending the observation time will increase the value of $P(A_{int}(t) > p)$ asymptotically to one. The reason for such behavior is due to the steady state availability of the system. From the analysis of the asymptotic behavior of the interval availability, it will converge to the value of steady state availability as $t \to \infty$. In other words, *with a sufficiently long observation period, $A_{int}(t) \approx A_{ss}$*. Therefore, for a guaranteed level of p given that $p < A_{ss}$, the probability of achieving at least p interval availability tends to one as $t \to \infty$. Nevertheless, when the specified guaranteed level p increases, the probability of achieving an interval availability of at least p becomes lower. For instance, when the length of the observation period is 22 time units, the probability of achieving at least $p = 0.7$ interval availability is 95%. This probability will be only 90% if $p = 0.8$.

2) When $p \geq A_{ss}$: If the guaranteed level $p \geq A_{ss}$, the behavior of the CCDF is disparate from the previously obtained results as illustrated in Fig. 17. In this case, $P(A_{int}(t) > p)$ is monotonically decreasing when the observation period increases, as observed in Fig. 17. The reason for this behavior is explained as follows. Since $A_{int}(t)$ converges to the steady state channel availability, A_{ss}, when $t \to \infty$, the probability of achieving an interval availability level which is higher than A_{ss} is lower. For small values of t, this probability is higher in contrast with the large values of t. This is because that the fraction of the accumulated time which the system is residing in channel available states is higher in the beginning. However this fraction goes down monotonically as we observe the system over a longer period. Therefore, $\lim_{t\to\infty} P(A_{int}(t) > p) = 0$ if $p \geq A_{ss}$.

J. Reliability Analysis in CRNs with Hybrid SU Traffic

In this subsection, we consider a heterogeneous SU traffic scenario which consists of two types of SU flows, i.e., elastic traffic and real-time traffic. While elastic flows adjust their transmission rate according to network conditions, real-time applications have the same session duration even though the data rate may vary from time to time. In this scenario, the number of channels allocated to a real-time SU (RSU) service is considered to be fixed to a single channel while an elastic SU (ESU) service can occupy from $W = 1$ up to $V = 3$ channels, the same as in the single-type SU scenario. An RSU service has higher priority than an ESU service if there is a need to release a channel to a PU arrival by reducing channel occupancy of an SU service or even forcibly terminating it. In addition, an ESU service can be preempted by an incoming RSU service if no idle channel exists upon an RSU arrival. Fig. 18 shows the steady state channel availability and mean channel available time for ESU and RSU services when the arrival rates (of ESU and RSU) and the service rates (of ESU and RSU) are set as follows: $\lambda_{ESU} = 2.0, \lambda_{RSU} = 1.5, \mu_{ESU} = 1.0$ and $\mu_{RSU} = 0.75$. Evidently, both the steady state channel availability and the mean channel available time decrease as the PU arrival rate becomes higher. However, the RSU services outperform the ESU services with respect to those reliability metrics since they have higher channel access privilege.

K. Reliability Analysis in CRNs with Heterogeneous Channels

Finally, we evaluate the performance of Scenario II introduced in Sec. III-D with $W = 1$ and $V = 3$. Consider now a CRN with $M_1 = 3$ and $M_2 = 6$. The channel allocation in our evaluation herein is selected in such a way that the total bandwidth for Scenario II is the same as in Scenario I with $M = 6$. More specifically, the bandwidth for a wideband channel is equal to the bandwidth of a channel in Scenario I while one channel in the NBC occupies half bandwidth of a channel in Scenario I. Correspondingly, the service rate per channel is configured as $\mu_{S1} = 1.0, \mu_{S2} = 0.5, \mu_{P1} = 0.5$ and $\mu_{P2} = 0.25$ services per unit of time respectively. In Fig. 19, we demonstrate three channel availability measures corresponding to the aforementioned heterogeneous channel environment. Comparing Fig. 19(a) with Fig. 7, we observe similar variations of the steady state channel availability in the heterogeneous channel scenario. Moreover, as illustrated in Fig. 19(b) and Fig. 19(c), $\bar{T}_{UT}$ and $\bar{T}_{FF}$ show also the same trend as depicted in Fig. 8 and Fig. 10 respectively. Therefore, the results shown in Fig. 19 clearly confirm the robustness and the applicability of the developed models in other kinds of network scenarios.

In order to compare the performance for both scenarios, we depict the steady state channel availability in Fig. 19(a) under both homogeneous and heterogeneous conditions. As we observe from this figure, subject to the same arrival and departure rate configurations, channel availability in Scenario II shows a higher value than that of Scenario I. The reason for this result is as follows. Consider a CRN with $M = 6$ homogeneous channels in Scenario I. If three out of the six channels are occupied, then only three channels are available for newly arrived services. Re-configure this CRN to the heterogeneous scenario with the same wideband and narrowband relationship mentioned above, we end up with a CRN with $M_1 = 3$ and $M_2 = 6$. This means that there are still six narrowband channels available in the CRN even if the three wideband channels are occupied. In other words, channel fragmentation leads to an increased number of channels. Consequently, the steady state channel availability is increased.

VII. Conclusions

The reliability and availability aspects of CRNs remain largely un-chartered despite tremendous research efforts within the area of CR during the past decade. In this paper, we define five reliability and availability metrics for channel access in both homogeneous and heterogeneous CRNs from the perspective of dependability theory and develop Markov chain based models to analyze these metrics. The mathematical expressions for those metrics as well as the distribution of channel availability are derived using the developed models. The numerical results show that the steady state channel availability, the mean time to channel unavailability and the mean channel available time decrease with a higher PU arrival rate and increase with a higher PU service rate. Furthermore the distribution of channel availability is derived based on a uniformization tool. We propose thereafter a measure to calculate the probability that the channel availability during the observation period t is not lower than a pre-defined guaranteed level p and evaluate it with respect to t and p. We believe that the definitions and the models presented in this paper collectively provide a systematic approach for availability analysis of channel access in multi-channel CRNs.

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TABLE III: Transitions from a generic state $\boldsymbol{x} = (i_{pu}, j_W, j_{W+1}, \cdots j_V)$ of DFA upon PU and SU events. The notations, AR and DP, indicate an arrival event and a departure event respectively.

Activity	Destination State	Tran. rate	Conditions
PU AR. A vacant channel exists.	$(i_{pu}+1, j_W, j_{W+1}, \cdots, j_k, \cdots, j_V)$	λ_P	$b(\boldsymbol{x}) < M$.
PU AR. No vacant channel exists. An SU with k channels reduces its aggregated channels.	$(i_{pu}+1, j_W, j_{W+1}, \cdots j_{k-1}+1, j_k - 1, 0, \cdots, 0)$	λ_P	$b(\boldsymbol{x}) = M; j_k > 0, k = \max\{r \mid j_r > 0, W+1 \le r \le V\}$.
PU AR. An SU service is terminated. No spectrum adaptation is executed.	$(i_{pu}+1, j_W - 1, j_{W+1}, \cdots, j_V)$	λ_P	$b(\boldsymbol{x}) = M$; (($j_W > 0, W = V$.) Or ($j_W \ge 1; W = 1, j_k = 0$ for $k > W+1$.) Or ($j_W = 1; j_k = 0,\ \forall k > W$.))
PU AR. An SU service is terminated and provides idle channels. Another SU service uses the idle channels.	$(i_{pu}+1, j_W - 2, 0, \cdots j_l, 0, \cdots, 0)$	λ_P	$b(\boldsymbol{x}) = M; j_W > 1; j_l = 1, l = 2W-1 \le V; W > 1; j_k = 0,\ \forall k > W$.
...	...	...	...
PU AR. An SU service is terminated. The remaining ongoing SU services use the idle channels and achieve the upper bound V.	$(i_{pu}+1, 0, \cdots, 0, j_V + q)$	λ_P	$b(\boldsymbol{x}) = M; q = j_W - 1; j_W > 1; W-1 \ge (V-W)(j_W - 1); V > W > 1$.
PU DP. An SU with k channels uses the idle channel.	$(i_{pu} - 1, 0, \cdots, 0, j_k - 1, j_{k+1} + 1, \cdots j_V)$	$i\mu_P$	$j_k > 0, k = \min\{r \mid j_r > 0, W \le r \le V-1\}; i_{pu} > 0; V > 1$.
PU DP. SUs cannot use the idle channel.	$(i_{pu} - 1, j_W, j_{W+1}, \cdots, j_k, \cdots j_V)$	$i\mu_P$	$V = W; i_{pu} > 0$. Or $j_k = 0,\ \forall k < V; i_{pu} > 0$.
SU AR. Enough idle channels exist.	$(i_{pu}, j_W, j_{W+1}, \cdots, j_k + 1, \cdots, j_V)$	λ_S	$k = \min\{M - b(\boldsymbol{x}), V\} \ge W$.
SU AR. The ongoing SU service with the maximum number of occupied channels, m, donates channels to the newly arrived service.	$(i_{pu}, j_W + 1, \cdots, j_n + 1, \cdots, j_m - 1, 0, \cdots, 0)$	λ_S	$M - b(\boldsymbol{x}) < M; m = \max\{r \mid j_r > 0, W+1 \le r \le V\}; n = m - [W - (M - b(\boldsymbol{x}))] \ge W; V > 1$.
SU AR. Two ongoing SU services using m and h channels respectively, donate channels to the newly arrived service.	$(i_{pu}, j_W + 2, \cdots, j_n + 1, \cdots, j_h - 1, \cdots, j_m - 1, 0, \cdots, 0)$	λ_S	$M - m = \max\{r \mid j_r > 0, W+1 \le r \le V\}; h = \max\{r \mid j_r > 0, W+1 \le r \le m-1\}$ if $j_m = 1$, Or $h = m$ if $j_m > 1; n = h + m - 2W + M - b(\boldsymbol{x}) \ge W; V > 1; W > 1$.
...	...	...	...
SU AR. All ongoing SU services that aggregate more than W channels donate channel(s) to the newly arrived SU service.	$(i_{pu}, j_W + q, \cdots, , j_n + 1, 0, \cdots, 0)$	λ_S	$q = \sum_{m=W+1}^{V} j_m; n = \sum_{m=W+1}^{V} (m-W) j_m + M - b(\boldsymbol{x}), W \le n < \min\{r \mid j_r > 0, W+1 \le r \le V\}; V > 1$.
SU with k channels DP, $j_k = 1$. Other SU services, if exist, cannot use the idle channel(s).	$(i_{pu}, 0, \cdots, 0, j_k - 1, 0, \cdots, 0, j_V)$	$k j_k \mu_S$	$j_k = 1, k < V, j_m = 0\ \forall m \ne k, V$.
SU with k channels DP, $j_k > 0$. Other SU services, if exist, cannot use the idle channel(s).	$(i_{pu}, 0, \cdots, 0, j_V - 1)$	$k j_k \mu_S$	$j_k > 0, k = V; j_m = 0,\ \forall m < V$.
SU with k channels DP. An SU service with minimum number of aggregated channels, h, uses all the idle channel(s).	$(i_{pu}, j_W, \cdots, j_h - 1, \cdots, j_k - 1, \cdots, j_l + 1, \cdots, j_V)$	$k j_k \mu_S$	$j_k > 1, h = \min\{r \mid j_r > 0, W \le r \le V-1\}; l = k + h \le V; V > 1$. Or $j_k = 1; h = \min\{r \mid j_r > 0, r \ne k, W \le r \le V-1\};\ l = k + h \le V; V > 1$.
...	...	...	...
SU with k channels DP. The remaining SU services with fewer than V channels use the idle channel(s) and achieve the upper bound V.	$(i_{pu}, 0, \cdots, 0, j_V + q)$	$k j_k \mu_S$	$q = \sum_{m=W}^{V-1} j_m - 1; k \ge \sum_{m=W}^{V-1} j_m (V - m); V > 1$.

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TABLE IV: Transitions from a generic state $\boldsymbol{x} = (i^w, j^w_W, j^w_{W+1}, \cdots, j^w_V, i^n, j^n_W, j^n_{W+1}, \cdots j^n_V)$ of DFA_H upon PU and SU events. The notations, AR and DP, indicate an arrival event and a departure event respectively. An SU service and a PU service in the NBC are denoted as SU_N and PU_N respectively. An SU service and a PU service in the WBC are denoted as SU_W and PU_W respectively.

Activity	Destination State	Tran. rate	Conditions
PU AR. A vacant channel exists in the WBC.	$(i^w+1, j^w_W, \cdots, j^w_V, i^n, j^n_W, j^n_{W+1}, \cdots j^n_V)$	λ_P	$b_w(\boldsymbol{x}) < M_1$.
PU AR. No vacant channel exists in the WBC. An SU_W with k channels reduces its aggregated channels.	$(i^w + 1, j^w_W, j^w_{W+1}, \cdots j^w_{k-1} + 1, j^w_k - 1, \cdots j^w_V, i^n, j^n_W, j^n_{W+1}, \cdots j^n_V)$	λ_P	$b_w(\boldsymbol{x}) = M_1; j^w_k > 0, k = \max\{r \mid j^w_r > 0, W+1 \leq r \leq V\}$.
PU AR. No vacant channel exists in the WBC. An SU_W with W channels performs spectrum handover to NBC.	$(i^w+1, j^w_W - 1, 0, \cdots, 0, i^n, 0, \cdots, 0, j^n_h + 1, \cdots j^n_V)$	λ_P	$b_w(\boldsymbol{x}) = M_1; j^w_W > 0, j^w_k = 0\ \forall k > W; h = \min\{M_2 - b_n(\boldsymbol{x}), V\} \geq W$.
PU AR. No vacant channel exists in the WBC. An SU_W with W channels performs spectrum handover to NBC and SU_N with k channels reduces its aggregated channels.	$(i^w + 1, j^w_W - 1, 0, \cdots, 0, i^n, j^n_W + 1, \cdots, j^n_h + 1, \cdots, j^n_m - 1, 0, \cdots, 0)$	λ_P	$b_w(\boldsymbol{x}) = M_1, b_n(\boldsymbol{x}) = M_2; j^w_W > 0, j^w_l = 0\ \forall l > W; m = \max\{r \mid j^n_r > 0, W+1 \leq r \leq V\}; h = m - W, W \leq h < m; V > 1$.
...	...	...	...
PU AR. No vacant channels exist in the CRN. An SU_W is forced to terminate.	$(i^w+1, j^w_W - 1, 0, \cdots, 0, i^n, j^n_W, 0, \cdots, 0)$	λ_P	$b_w(\boldsymbol{x}) = M_1, b_n(\boldsymbol{x}) = M_2; j^w_W > 0; j^w_k = 0\ \forall k > W; j^n_h = 0\ \forall h > W$.
PU AR. A vacant channel exists in the NBC.	$(i^w, 0, \cdots, 0, i^n + 1, j^n_W, \cdots, j^n_V)$	λ_P	$b_w(\boldsymbol{x}) = M_1; j^w_k = 0\ \forall k \geq W; b_n(\boldsymbol{x}) < M_2$.
PU AR. No vacant channel exists in the CRN. An SU_N with k channels reduces its aggregated channels.	$(i^w, 0, \cdots, 0, i^n+1, j^n_W, \cdots j^n_{k-1}+1, j^n_k - 1, 0, \cdots, 0)$	λ_P	$b_w(\boldsymbol{x}) = M_1, b_n(\boldsymbol{x}) = M_2; j^w_k = 0\ \forall k \geq W; j^n_k > 0; k = \max\{r \mid j^n_r > 0, W+1 \leq r \leq V\}$.
PU AR. No vacant channels exist in the CRN. An SU_N is forced to terminate.	$(i^w, 0, \cdots, 0, i^n + 1, j^n_W - 1, 0, \cdots, 0)$	λ_P	$b_w(\boldsymbol{x}) = M_1, b_n(\boldsymbol{x}) = M_2; j^w_k = 0\ \forall k \geq W; j^n_W > 0, j^n_h = 0\ \forall h > W$.
PU_W DP. A PU_N performs spectrum handover to the WBC. An SU_N with k channels aggregates the idle channel.	$(i^w, j^w_W, j^w_{W+1}, \cdots j^w_V, i^n - 1, 0, \cdots, 0, j^n_k - 1, j^n_{k+1} + 1, \cdots j^n_V)$	$i^w \mu_{P1}$	$i^w > 0, i^n > 0; j^n_k > 0, k = \min\{r \mid j^n_r > 0, W \leq r \leq V-1\}$.
PU_W DP. An SU_N performs spectrum handover to the WBC. An SU_N with k channels aggregates the idle channel.	$(i^w - 1, j^w_W + 1, j^w_{W+1}, \cdots j^w_V, i^n, j^n_W - 1, 0, \cdots, 0, j^n_k - 1, j^n_{k+1} + 1, \cdots j^n_V)$	$i^w \mu_{P1}$	$i^w > 0, i^n = 0; j^n_W > 0, W = 1; j^n_k > 0, k = \min\{r \mid j^n_r > 0, W \leq r \leq V-1\}$.
PU_W DP. An SU_W with k channels aggregates the idle channel.	$(i^w - 1, 0, \cdots, 0, j^w_k - 1, j^w_{k+1} + 1, \cdots j^w_V, i^n, j^n_W, j^n_{W+1}, \cdots j^n_V)$	$i^w \mu_{P1}$	$i^w > 0, i^n = 0; j^n_W = j^n_{W+1} = 0, W = 1; j^w_k > 0, k = \min\{r \mid j^w_r > 0, W \leq r \leq V-1\}$.
PU_N DP. An SU_N with k channels aggregates the idle channel.	$(i^w, j^w_W, j^w_{W+1}, \cdots j^w_V, i^n - 1, 0, \cdots, 0, j^n_k - 1, j^n_{k+1} + 1, \cdots j^n_V)$	$i^n \mu_{P2}$	$i^n > 0; j^n_k > 0, k = \min\{r \mid j^n_r > 0, W \leq r \leq V-1\}$.
SU AR. Enough idle channels exist in the WBC.	$(i^w, j^w_W, j^w_{W+1}, \cdots j^w_k + 1, j^w_{k+1}, \cdots j^w_V, i^n, j^n_W, j^n_{W+1}, \cdots j^n_V)$	λ_S	$k = \min\{M_1 - b_w(\boldsymbol{x}), V\} \geq W$.
SU AR. SU_W with the maximum number of occupied channels, m, donates channel(s) to the newly arrived service.	$(i^w, j^w_W + 1, j^w_{W+1}, \cdots, j^w_n + 1, \cdots, j^w_m - 1, 0, \cdots, 0, i^n, j^n_W, j^n_{W+1}, \cdots, j^n_V)$	λ_S	$b_w(\boldsymbol{x}) = M_1; m = \max\{r \mid j^w_r > 0, W+1 \leq r \leq V\}; n = m - [W - (M_1 - b_w(\boldsymbol{x}))], W \leq n < m; V > 1$.
SU AR. Enough idle channels exist in the NBC.	$(i^w, j^w_W, 0, \cdots, 0, i^n, j^n_W, j^n_{W+1}, \cdots, j^n_k + 1, \cdots j^n_V)$	λ_S	$b_w(\boldsymbol{x}) = M_1; j^w_l = 0\ \forall l > W; k = \min\{M_2 - b_n(\boldsymbol{x}), V\} \geq W$.
SU AR. SU_N with the maximum number of occupied channels, m, donates channel(s) to the newly arrived service.	$(i^w, j^w_W, 0, \cdots, 0, i^n, j^n_W + 1, j^n_{W+1}, \cdots j^n_n + 1, \cdots, j^n_m - 1, 0, \cdots, 0)$	λ_S	$b_w(\boldsymbol{x}) = M_1, b_n(\boldsymbol{x}) = M_2; j^w_l = 0\ \forall l > W; m = \max\{r \mid j^n_r > 0, W+1 \leq r \leq V\}; n = m - W, W \leq n < m; V > 1$.
...	...	...	...
SU AR. All ongoing SU_N services that aggregate more than W channels donate channel(s) to the newly arrived SU service.	$(i^w, j^w_W, 0, \cdots, 0, i^n, j^n_W + q, 0, \cdots j^n_n + 1, 0, \cdots, 0)$	λ_S	$b_w(\boldsymbol{x}) = M_1, b_n(\boldsymbol{x}) = M_2; j^w_l = 0\ \forall l > W; q = \sum_{m=W+1}^{V} j^n_m; n = \sum_{m=W+1}^{V} (m - W) j^n_m + M_2 - b_n(\boldsymbol{x}), W \leq n < \min\{r \mid j^n_r > 0, W+1 \leq r \leq V\}; V > 1$.
SU_W with k channels DP. Other SU services, if exist, cannot use the idle channel(s).	$(i^w, j^w_W, j^w_{W+1}, \cdots j^w_k - 1, j^w_{k+1}, \cdots j^w_V, i^n, j^n_W, j^n_{W+1}, \cdots j^n_V)$	$k j^w_k \mu_{S1}$	$(j^w_k = 1, k < V; j^w_h = 0\ \forall h < V$ and $h \neq k)$ Or $(j^w_k > 0, k = V; j^w_h = 0\ \forall h < V); j^n_m = 0\ \forall m < 2k$.
SU_W with k channels DP. An SU_N with l channels performs spectrum handover to the WBC. An SU_N with h channels aggregates the idle channel.	$(i^w, j^w_W, \cdots j^w_k, \cdots j^w_V, i^n, j^n_W, \cdots, j^n_l - 1, \cdots, j^n_h - 1, \cdots, j^n_q + 1, \cdots j^n_V)$	$k j^w_k \mu_{S1}$	$(j^n_l > 1, l = \min\{r \mid j^n_r > 0, W \leq r < V\}; h = l)$ Or $(j^n_l = 1, l = \min\{r \mid j^n_r > 0, W \leq r < V\}; h = \min\{r \mid j^n_r > 0, r > l\}); q = h + l \leq V; V > 1$.
SU_W with k channels DP. An SU_W with minimum number of aggregated channels, h, uses all the idle channel(s).	$(i^w, j^w_W, \cdots j^w_h - 1, \cdots j^w_k - 1, \cdots j^w_l + 1, \cdots j^w_V, i^n, j^n_W, j^n_{W+1}, \cdots j^n_V)$	$k j^w_k \mu_{S1}$	$j^w_k > 0, j^n_m = 0\ \forall m < 2k; h = \min\{r \mid j^w_r > 0, W \leq r \leq V-1\}; l = k + h \leq V; V < 1$.
...	...	...	...
SU_W with k channels DP. All other SU_W services with fewer than V channels use the idle channel(s) and achieve the upper bound V.	$(i^w, 0, \cdots, 0, j^w_V + q, i^n, j^n_W, j^n_{W+1}, \cdots j^n_V)$	$k j^w_k \mu_{S1}$	$j^n_l = 0\ \forall l < 2k; q = \sum_{m=W}^{V-1} j^w_m - 1; k \geq \sum_{m=W}^{V-1} (V - m) j^w_m - (V - k); V > 1$
SU_N with k channels DP. Other SU services, if exist, cannot use the idle channel(s).	$(i^w, j^w_W, j^w_{W+1}, \cdots j^w_V, i^n, j^n_W, \cdots, j^n_k - 1, j^n_{k+1}, \cdots j^n_V)$	$k j^n_k \mu_{S2}$	$j^n_k = 1, j^n_m = 0\ \forall m < V$ and $m \neq k$. Or $j^n_k > 0, k = V; j^n_m = 0\ \forall m < V$.
SU_N with k channels DP. An SU_N with minimum number of aggregated channels, h, uses all the idle channel(s).	$(i^w, j^w_W, j^w_{W+1}, \cdots j^w_V, i^n, j^n_W, \cdots j^n_h - 1, \cdots j^n_k - 1, j^n_{k+1}, \cdots, j^n_l + 1, \cdots, j^n_V)$	$k j^n_k \mu_{S2}$	$j^n_k > 0; h = \min\{r \mid j^n_r > 0, W \leq r \leq V-1\}; l = k + h \leq V; V > 1$.
...	...	...	...
SU_N with k channels DP. The remaining SU_N services with fewer than V channels use the idle channel(s) and achieve the upper bound V.	$(i^w, j^w_W, j^w_{W+1}, \cdots j^w_V, i^n, 0, \cdots, 0, j^n_V + q)$	$k j^n_k \mu_{S2}$	$q = \sum_{m=W}^{V-1} j^n_m - 1; k \geq \sum_{m=W}^{V-1} (V - m) j^n_m - (V - k); V > 1$

APPENDIX
CALCULATION OF $\bar{T}_{DT}$ IN A SINGLE CHANNEL CRN

Consider a CRN with a single channel, i.e., $M = 1$ and a simple channel access scheme where SUs opportunistically access the channel while PUs have the priority for channel access. Assuming Markovian PU/SU arrivals and departures, the states of the corresponding CTMC model are represented by $\boldsymbol{x} = (i_{pu}, j_{su})$ where i_{pu} and j_{su} respectively denote the number of PU and SU services in the CRN. The feasible state space of this system is represented by $\mathcal{S} = \{(0,0),(0,1),(1,0)\}$. A new SU arrival will access the CRN only when the system is in state $(0,0)$. Therefore, $\mathcal{S}_{\mathcal{A}} = \{(0,0)\}$ and $\mathcal{S}_{\mathcal{B}} = \{(0,1),(1,0)\}$ where $\mathcal{S}_{\mathcal{A}}$ and $\mathcal{S}_{\mathcal{B}}$ denote the set of channel available and unavailable states respectively. Accordingly, the transition rate matrix of this network is constructed as

$$\begin{array}{cc} & \begin{array}{ccc} \text{States} & (0,0) & (0,1) \quad (1,0) \end{array} \\ \boldsymbol{Q} = & \begin{array}{c} (0,0) \\ (0,1) \\ (1,0) \end{array} \begin{pmatrix} -(\lambda_S + \lambda_P) & \lambda_S & \lambda_P \\ \mu_S & -(\mu_S + \lambda_P) & \lambda_P \\ \mu_P & 0 & -\mu_P \end{pmatrix}. \end{array}$$

Assume that $\lambda_P = \lambda_S = 2$ and $\mu_P = 0.5, \mu_s = 1$. We have

$$\boldsymbol{Q} = \begin{bmatrix} -4 & 2 & 2 \\ 1 & -3 & 2 \\ 0.5 & 0 & -0.5 \end{bmatrix}.$$

Since state $(0,0)$ is the only channel available state in this example, the sub-matrices in the partitioned form in (5) are obtained respectively as follows

$$\boldsymbol{A} = \begin{bmatrix} -4 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 2 & 2 \end{bmatrix},$$
$$\boldsymbol{C} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} -3 & 2 \\ 0 & -0.5 \end{bmatrix}.$$

The steady state probability vectors are obtained by solving (4) and they are given as $\boldsymbol{\pi}_A = \begin{bmatrix} 0.12 \end{bmatrix}$ and $\boldsymbol{\pi}_B = \begin{bmatrix} 0.08 & 0.8 \end{bmatrix}$ for channel available states and channel unavailable states respectively. Once we reach this step, all necessary parameters that are needed to obtain the values of the performance metrics given in Sec. IV-C have been obtained. For instance, substituting these values into (16) yields the *mean channel unavailable time* as follows:

$$\begin{aligned} \bar{T}_{DT} &= \frac{\boldsymbol{\pi}_B \boldsymbol{U}_{N-L}}{\boldsymbol{\pi}_A \boldsymbol{B} \boldsymbol{U}_{N-L}} \\ &= \frac{\begin{bmatrix} 0.08 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 0.12 \end{bmatrix} \begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}} = \frac{0.88}{0.12 \cdot 4} = 1.8333 \text{ time units.} \end{aligned}$$

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