

Document downloaded from:

<http://hdl.handle.net/10251/99645>

This paper must be cited as:

Camacho Vidal, FJ.; Cortés, J.; Micle \*, RM.; Sánchez-Sánchez, A. (2013). Predicting the academic underachievement in high school in Spain over the next few years: A dynamic modelling approach. *Mathematical and Computer Modelling*. 57(7):1703-1708.  
doi:10.1016/j.mcm.2011.11.011



The final publication is available at

<http://doi.org/10.1016/j.mcm.2011.11.011>

Copyright Elsevier

Additional Information

# Predicting the academic underachievement in high school in Spain over the next few years: A dynamic modelling approach

J. Camacho, J.-C. Cortés\*, R.-M. Micle, A. Sánchez-Sánchez

*Instituto Universitario de Matemática Multidisciplinar  
Universitat Politècnica de València  
Building 8G, 2<sup>nd</sup> floor, Access C, 46022 Valencia (Spain)*

---

## Abstract

In this paper we propose a dynamic model to understand the evolution of the academic underachievement in high school in Spain. This model is based on ideas of Christakis and Fowler where individual habits may be transmitted by social contact. Thus, to build the model we suppose that a student has academic failure when she/he gets into study habits transmitted by students with bad academic habits. From the available academic results of the Spanish high school educational system during the period 1999/2008, we fit the model to the data in order to obtain the parameters of the model. Then, we predict the academic underachievement evolution over the next few years.

*Key words:* epidemiological-type model, academic underachievement, evolution dynamics, prediction

---

## 1 Introduction

During the last few years, Spanish academic authorities have taken a number of educational measures in order to combat academic underachievement in high schools including an increase in funding and resources destined to education improvement [1]. As a result of these education policies, the percentage of students with academic underachievement in the final stage of Spanish high

---

\* Corresponding author

*Email addresses:* `fcamacho@mat.upv.es` (J. Camacho), `jccortes@imm.upv.es` (J.-C. Cortés), `ramic@ade.upv.es` (R.-M. Micle), `alsncsnc@posgrado.upv.es` (A. Sánchez-Sánchez).

school, called *Bachillerato*, certainly has reduced but just timidly at levels still very worrying. In fact, the vertical bars that correspond to  $\overline{W}_1$ ,  $\overline{M}_1$ ,  $\overline{W}_2$  and  $\overline{M}_2$  in the plots of Figure 3 show that the total percentage of students that do not pass in this educational level has stabilized around 30%. This high rate of academic underachievement in Spain is becoming a major social and political concern because it is a decisive factor for unemployment and also influences the future development of a country. The analysis of school performance has an important role in achieving quality education and increasing its efficiency. The number of students that abandon their studies not only in the stage of *Bachillerato* but also in other educational levels, and therefore they are not going to be qualified workers is greater than the number of qualified jobs expected over the following years. This will lead to an increase in unemployment among young students, which is also a primer concern of the European Union [2].

We focus our attention on the data from the Spanish *Bachillerato* that theoretically correspond to students who are 16–18 years old. They come from Compulsory Secondary Education, called *Educación Secundaria Obligatoria* (ESO), where students should stay for four years (from 12 to 16 years old). After finishing with this academic level, a great part of them decide to access to *Bachillerato*, which is not mandatory. *Bachillerato* consists of two years and the subjects are more specialized in science, literature or art than in ESO. The main objective of *Bachillerato* training is to prepare high school students for university studies, although students passing to the *Bachillerato* level can decide to access directly to the labor market or to continue non-university training studies (see Figure 1). In accordance with educational regulations in force in Spain (called LOE), a student of first stage of *Bachillerato* passes to the second stage of *Bachillerato*, if at most she/he fails two subjects out of ten that has the full course [3,4]. As the second stage of *Bachillerato* constitutes the last course of this educational level, a student will promote this academic level if she/he passes all the subjects.

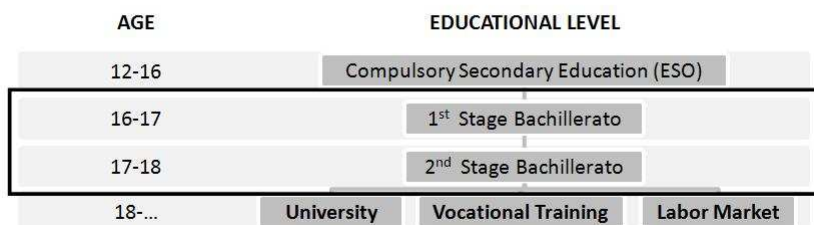


Figure 1. Structure of the Spanish educational system for students aged between 12–18 years old.

In this paper we are interested in studying the academic underachievement in *Bachillerato*. To address this problem, in addition to consider autonomous

behavior of students who take decisions about their own priorities, goals, etc., we assume that academic underachievement is a socially transmitted behavior [5,6]. We treat school failure as a problem that is transmitted through social contact. These social contacts have an influence on the probability of transmission of bad study habits. The main idea behind this approach is that these inappropriate habits may spread from one student to another, more probably between students of the same academic level. These facts lead us to propose an epidemiological-type mathematical model to study the evolution (transmission dynamics) of the academic underachievement in *Bachillerato*. According to [7], there exists a significative difference of academic performance depending on genre. In general, women obtain better academic results than men. This motivates the fact that our model considers genre into its formulation.

We focus our attention on *Bachillerato* educational stage for several reasons. Firstly, to the best of our knowledge, none study based on the previous epidemiological approach has been proposed to study academic underachievement. From a mathematical standpoint, *Bachillerato* has a simpler academical structure that seems to be an adequate start-point to introduce these type of modeling approach. Secondly, from a social viewpoint, *Bachillerato* represents a milestone in the career training of students because, as we said, when they finish *Bachillerato*, they can decide whether to continue their higher studies (university or professional training) or to access the work market. This is of paramount importance for society because as we pointed out previously, although the percentage of high school academic underachievement has reduced over the last years, nowadays it seems to be at a worrying steady-level. This constitutes a serious problem not only for these individuals and their families but also for the society that has invested an important amount of money in their previous training. Thus, the construction of an adequate mathematical model allows us to understand the mechanism behind the academic performance and to predict how things will evolve over the next few years.

## 2 Building the mathematical model

### 2.1 Available data

In this paper, we have considered the available data corresponding to the first and second stage of *Bachillerato*, in both, public and private high schools all over Spain during academic years 1999/2000 to 2007/2008. We point out that we study this period because after the academic year 2007/2008 only partial data corresponding to specific regions of Spain are available. For each of these academic years, Table 1 collects the percentage of women/men that pass ( $W_i/M_i$ ) and do not pass ( $\bar{W}_i/\bar{M}_i$ ) for each level  $i = 1, 2$  (that corresponds to

the first and second stage of *Bachillerato*, respectively) over the total Spanish *Bachillerato* students. After some algebraic manipulations, these data have been obtained from the official database [8,9] and they are referred to the month of September, when each academic year ends officially.

Table 1

The available data corresponding to the first and second stage of *Bachillerato*, in both, public and private high schools all over Spain during academic years 1999/2000 to 2007/2008. Each row shows the percentage of women/men that promote ( $W_i/M_i$ ) and do not promote ( $\overline{W}_i/\overline{M}_i$ ) for each level  $i = 1, 2$  (that corresponds to the first and second stage of *Bachillerato*, respectively) over the total Spanish *Bachillerato* students. These data are referred to the month of September, when each academic year ends officially.

Academic year	First stage of Bachillerato (Women/Men)		Second stage of Bachillerato (Women/Men)	
	% Pass ( $W_1/M_1$ )	% Do not Pass ( $\overline{W}_1/\overline{M}_1$ )	% Pass ( $W_2/M_2$ )	% Do not Pass ( $\overline{W}_2/\overline{M}_2$ )
1999-00	19.68/15.24	9.75/9.33	16.21/11.64	9.52/8.63
2000-01	22.65/17.54	9.91/10.12	14.07/10.04	8.24/7.43
2001-02	19.23/14.23	8.61/9.10	17.86/13.06	9.32/8.59
2002-03	18.87/14.19	8.36/8.51	19.14/13.97	8.76/8.20
2003-04	19.93/15.06	7.74/7.88	19.19/13.80	8.44/7.96
2004-05	20.11/15.14	7.65/7.94	18.90/13.92	8.39/7.95
2005-06	20.07/15.39	7.64/7.93	19.14/13.97	8.08/7.78
2006-07	20.06/15.34	7.67/7.87	19.14/14.29	7.98/7.65
2007-08	20.25/15.82	7.57/7.66	19.37/14.61	7.60/7.12

Before describing the proposed model, we want to emphasize that other tentative models were discarded until its current formulation. For the sake of clarity, comments about this issue will be included throughout the paper.

We build our mathematical model based on an epidemiological-type model by considering that academic underachievement is a process that takes place when a female (W) or a male (M) student that initially belongs to the promotable group of a specific level,  $W_i$  or  $M_i$ ,  $i = 1, 2$ , leaves her/his good academic habits due to the negative influence (*contagion*) from other students of the *same* educational level who belong to the group of students with academic underachievement,  $\overline{W}_i$  and  $\overline{M}_i$ . We emphasize that, in spite of the fact that Spanish *Bachillerato* students share the same educational center with students belonging to other lower educational levels, it is realistic to assume that from an academic performance point of view, in general, the *contagion* between them is not significative. This statement is justified by the differences in age and maturity between students that belong to different educational levels, their physical location in the high school, etc. In the model, these considerations will be taken into account even for *Bachillerato* students of different academic level. We shall consider that, for each specific academic level under study, the bad academic habits just spread between students of the same course, independently of their genre. We point out that during the formulation of our model, we considered the transmission of bad study habits between students belonging to different stages of *Bachillerato* as a possibility, however

it was discarded because it implies an increasing of the number of parameters to be computed that, as we checked, it does not provide any better fitting. Thus, the transitions described can be modeled as follows:

- For a specific *Bachillerato* academic level  $i = 1, 2$ , a student in  $W_i$  (respec.  $M_i$ ) transits to  $\overline{W}_i$  (respec.  $\overline{M}_i$ ) because students in  $\overline{W}_i$  and  $\overline{M}_i$  transmit their negative academic habits at rates  $\beta_i^W$  (respec.  $\beta_i^M$ ). Therefore, this is a nonlinear term modeled by  $\beta_i^W W_i(\overline{W}_i + \overline{M}_i)$  (respec.  $\beta_i^M M_i(\overline{W}_i + \overline{M}_i)$ ). Note that modeling above assumes implicitly population mixing for each academic level under consideration. This is a usual assumption in type-continuous epidemiological models [10]. Note that encounters between all Spanish students of the same academic level are assumed implicitly through population mixing hypothesis which is not realistic, however coefficients  $\beta_i^W$  and  $\beta_i^M$  modulate themselves the weighting of these encounters.
- For a specific *Bachillerato* academic level  $i = 1, 2$ , students can also acquire bad academic habits because they autonomously decide to strive less and to give up appropriate study habits due to lack of self-motivation, personal problems, etc. This individual behavior can even be determined by external influences of other people including other students but being of different academic stage. This type of transition is modeled by means of a linear term  $\alpha_i^W W_i$  (respec.  $\alpha_i^M M_i$ ).
- Data collected in Table 1 refer to the end of each academic course, i.e., in September when, according to educational regulation in force in Spain, every student in  $W_1$  and  $M_1$  will pass to  $W_2$  and  $M_2$ , respectively. Taking into account that time  $t$  is measured in years and we identify each academic year in the period 1999/2000 to 2007/2008 with  $0, 1, \dots, 8$ , respectively, parameter  $\delta$  indicated in Figure 2 is defined as follows:

$$\delta = \begin{cases} 1 & \text{if } \frac{9}{12} + j \leq t \leq \frac{10}{12} + j, \quad j = 0, 1, 2, \dots, 8, \\ 0 & \text{otherwise.} \end{cases}$$

- For a specific *Bachillerato* academic level  $i = 1, 2$ , a student in  $\overline{W}_i$  (respec.  $\overline{M}_i$ ) transits to  $W_i$  (respec.  $M_i$ ), when she/he gives up her/his bad academic habit. An individual in  $\overline{W}_i$  (respec.  $\overline{M}_i$ ) transits to  $W_i$  (respec.  $M_i$ ) at rate  $\gamma_i^W$  (respec.  $\gamma_i^M$ ) proportionally to the size of  $\overline{W}_i$  (respec.  $\overline{M}_i$ ). Analogously to  $\alpha_i^W$  and  $\alpha_i^M$ , parameters  $\gamma_i^W$  and  $\gamma_i^M$  also contain those autonomous decisions adopted by students belonging to  $\overline{W}_i$  and  $\overline{M}_i$ , respectively.

Then, the transitions between these different subpopulations are described by the following coupled nonlinear system of differential equations where the unknowns are  $W_i = W_i(t)$ ,  $M_i = M_i(t)$ ,  $\overline{W}_i = \overline{W}_i(t)$  and  $\overline{M}_i = \overline{M}_i(t)$  ( $t$  denotes

time in years),

$$\begin{aligned}
W_1'(t) &= -\delta W_1(t) - \alpha_1^W W_1(t) - \beta_1^W W_1(t)[\overline{W}_1(t) + \overline{M}_1(t)] + \gamma_1^W \overline{W}_1(t), \\
\overline{W}_1'(t) &= \alpha_1^W W_1(t) + \beta_1^W W_1(t)[\overline{W}_1(t) + \overline{M}_1(t)] - \gamma_1^W \overline{W}_1(t), \\
W_2'(t) &= \delta W_1(t) - \alpha_2^W W_2(t) - \beta_2^W W_2(t)[\overline{W}_2(t) + \overline{M}_2(t)] + \gamma_2^W \overline{W}_2(t), \\
\overline{W}_2'(t) &= \alpha_2^W W_2(t) + \beta_2^W W_2(t)[\overline{W}_2(t) + \overline{M}_2(t)] - \gamma_2^W \overline{W}_2(t), \\
M_1'(t) &= -\delta M_1(t) - \alpha_1^M M_1(t) - \beta_1^M M_1(t)[\overline{W}_1(t) + \overline{M}_1(t)] + \gamma_1^M \overline{M}_1(t), \\
\overline{M}_1'(t) &= \alpha_1^M M_1(t) + \beta_1^M M_1(t)[\overline{W}_1(t) + \overline{M}_1(t)] - \gamma_1^M \overline{M}_1(t), \\
M_2'(t) &= \delta M_1(t) - \alpha_2^M M_2(t) - \beta_2^M M_2(t)[\overline{W}_2(t) + \overline{M}_2(t)] + \gamma_2^M \overline{M}_2(t), \\
\overline{M}_2'(t) &= \alpha_2^M M_2(t) + \beta_2^M M_2(t)[\overline{W}_2(t) + \overline{M}_2(t)] - \gamma_2^M \overline{M}_2(t).
\end{aligned} \tag{1}$$

The parameters of the model are:

- $\alpha_i^g$ , denotes the rate at which a student of Spanish *Bachillerato* academic level  $i$  and genre  $g$  who belongs to the promoting group passes to have bad academic habits by an autonomous decision. In accordance with personal trait patterns and academic performance of adolescents [11], it is considered that for the same educational stage, women are more responsible with respect to studies matters than men. Whereas comparing the same genre, students that belong to the second stage of *Bachillerato* get more involved than students of the first stage due not only to maturity but also they feel more self-motivated because they are about to get the *Bachillerato* degree. In consequence, We consider these academic behavioral conclusions by assuming, respectively, that

$$\alpha_1^W < \alpha_1^M, \quad \alpha_2^W < \alpha_2^M; \quad \alpha_1^W > \alpha_2^W, \quad \alpha_1^M > \alpha_2^M. \tag{2}$$

At this point, we notice that when we were constructing the model and, as a tentative purpose, we also tried its formulation without assuming restriction (2). However it was discarded because it implied an increase of the computational time without improving the model fitting.

- $\beta_i^g$ , denotes the transmission rate at which a student of Spanish *Bachillerato* academic level  $i$  and genre  $g$  adopts bad academic habits due to the negative influence from students that do not pass and belong to the same academic level  $i$  including both genre.
- $\gamma_i^g$ , denotes the rate at which a student of Spanish *Bachillerato* academic level  $i$  and genre  $g$  who has bad academic habits, by an autonomous decision, decides to change her/his bad academic habits and she/he ends up getting into the passing group.

The flow diagram, associated to the model, is depicted in Figure 2.

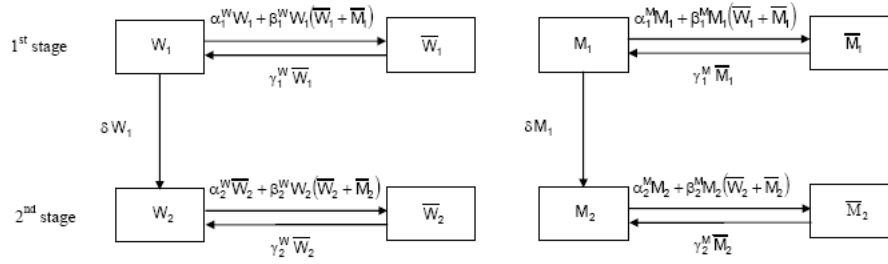


Figure 2. Flow diagram of the mathematical model for *Bachillerato* academic underachievement in Spain. The boxes represent subpopulations under study classified according to genre (women (W) and men (M)) and academic level (first and second stage of *Bachillerato*). Students that belong to the promoting/no promoting group are denoted by  $W$  and  $M$  /  $\bar{W}$  and  $\bar{M}$ , respectively. The arrows represent the transitions between the subpopulations, and they are labeled by their corresponding terms and parameters according to the model.

### 3 Parameter estimation

In this section we will estimate the parameters of (1) by fitting the model in the mean square sense to the data collected in Table 1. We have carried out computations with *Mathematica 7.0* [12]. As initial conditions of the system of differential equations (1), we take data of the academic year 1999/2000 (corresponding to  $t = 0$ ), so  $W_1(0) = 19.68$ ,  $M_1(0) = 15.24$ ,  $\bar{W}_1(0) = 9.75$ ,  $\bar{M}_1(0) = 9.33$ ,  $W_2(0) = 16.21$ ,  $M_2(0) = 11.64$ ,  $\bar{W}_2(0) = 9.52$  and  $\bar{M}_2(0) = 8.63$ . The estimated parameters are collected in Table 2. The value of the function in the global minimum (i.e., the least square error) that we have obtained is 0.0095.

Table 2  
Estimation of the model parameters

Parameter	Value	Parameter	Value	Parameter	Value
$\alpha_1^W$	0.0180156	$\beta_1^W$	0.538867	$\gamma_1^W$	0.285557
$\alpha_1^M$	0.0180175	$\beta_1^M$	2.59115	$\gamma_1^M$	0.853703
$\alpha_2^W$	0.000119175	$\beta_2^W$	0.668998	$\gamma_2^W$	0.270119
$\alpha_2^M$	0.00307025	$\beta_2^M$	1.38138	$\gamma_2^M$	0.405319

Regarding Table 2, for both educational stages, the transmission rate at which male students of *Bachillerato* adopt bad academic habits due to influence of students that do not pass is greater than the corresponding one for female students, i.e.,  $\beta_1^M > \beta_1^W$  and  $\beta_2^M > \beta_2^W$ . Whereas the rate at which a male student of *Bachillerato* who has bad academic habits decides autonomously to change his bad academic habits and he ends up getting into the passing group is greater than the corresponding one for a female student, i.e.,  $\gamma_1^M > \gamma_1^W$  and



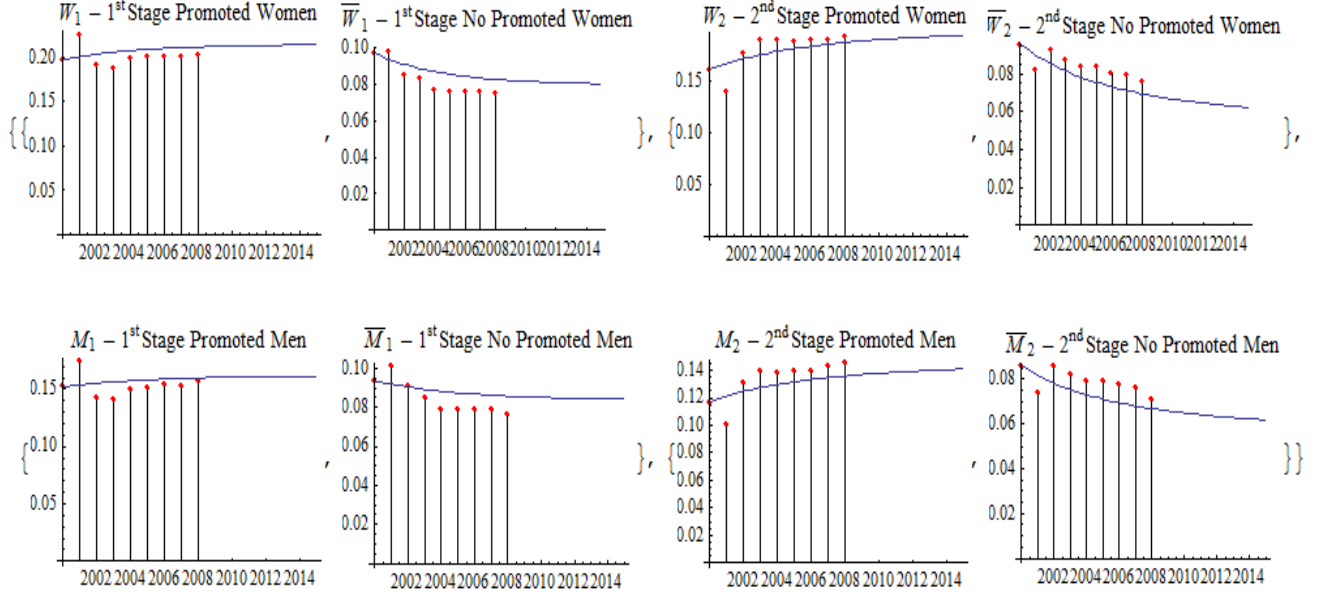


Figure 3. Fitting and prediction of the academic performance of *Bachillerato* Spanish students over the academic years 1999/2000 to 2014/2015

$\gamma_2^M > \gamma_2^W$ . We point out that both results agree with conclusions derived from other pedagogical studies [11].

#### 4 Prediction over next few years

Now, once the model is stated and the parameters estimated, we are able to give predictions of each subpopulation over the next few years computing the solutions of the model,  $W_1(t)$ ,  $\bar{W}_1(t)$ ,  $M_1(t)$ ,  $\bar{M}_1(t)$ ,  $W_2(t)$ ,  $\bar{W}_2(t)$ ,  $M_2(t)$  and  $\bar{M}_2(t)$  for values of time  $t$  in the future. The solution to model (1) is plotted in Figure 3.

According to the model predictions, the percentage of students that pass (do not pass) in the second stage of *Bachillerato* seems to increase (decrease) slightly as time goes. While in the first stage of *Bachillerato* the situation is different. In this case, the model forecasts a stabilization of the percentage of students that pass and do not pass. Regarding the global prediction of the model, note that the total percentage of *Bachillerato* students that will not pass is worrying because it will lie around 30% which still constitutes a high rate. In accordance with the model, the first stage of *Bachillerato* is the key level to begin to combat academic underachievement in *Bachillerato* not only because it has greater associated academic underachievement rates than second stage but also these rates tend to stabilization. From a pedagogical point of view this feature can be explained due to the considerable degree of difficulty of *Bachillerato* subjects with respect to ESO contents.

## 5 Conclusions

In this paper we have developed a continuous model to study academic underachievement in the last educational stage of the Spanish high school, called *Bachillerato*. The major novelty of this contribution is the treatment of school failure as a problem that is transmitted through social contact including its mathematical type-epidemiological modeling. We point out that the results we have obtained agree with conclusions derived from other studies but in addition the model allows seeing how things will evolve in the next future. Hence, it enables us to make an estimation of the number of pupils who will apply for higher education. This is a key issue since academic underachievement is a decisive factor for unemployment and also has a great influence in the future development of a country.

Finally, we want to point out that this contribution constitutes a first step in the modeling of academic underachievement applying a type-epidemiological approach. Therefore, additional studies must be done in the forthcoming work. On the one hand, based on similar ideas developed in [13] to excess weight health concern, we firstly plan to study the parameters of the model (1) in depth in order to reveal not only their marginal influence over the final results by means of a sensibility analysis but also to set suitable educational policies to improve the current situation. On the other hand, the treatment of model (1) as a system of random differential equations is more realistic since owing to the complexity of the phenomenon under study, in practice, both initial conditions and coefficients are not known in a deterministic way. Notice that our model does not consider noncompletion of Spanish *Bachillerato* studies because at the moment the available data do not provide this information. Although the abandon rates referred to the Spanish *Bachillerato* do not seem to be significant, in the future it would be desirable to reformulate the model (1) taking into account this factor and to contrast with our current proposal, whenever data are available. To conclude, notice that the formulation of models like (1) but adapted to another educational levels is also demanded.

**Acknowledgement:** This work has been partially supported by the Spanish M.C.Y.T. grant MTM2009-08587 and the Universitat Politècnica de València grant PAID06-09-2588.

## References

- [1] M.J. San Segundo, A. Vaquero, Descentralización educativa y programas nacionales de mejora (Decentralization of education and national improvement programs), XVI Jornadas de la Asociación de la Economía de la Educación, 1–17, 2008. (In Spanish).

- [2] <http://www.euractiv.com/en/enterprise-jobs/eu-youth-job-strategy-under-fire-news-497858> (Accessed on 12<sup>th</sup> of September 2011).
- [3] Real Decreto 1467/2007, de 2 de noviembre por el que se establece la estructura del bachillerato español y se fijan sus enseñanzas mínimas. (Law 1467/2007 of 2 November for establishing the structure of Spanish high school and set their minimum education) Available at <http://www.educacion.es/educacion/que-estudiar/bachillerato.html> (Accessed on 12<sup>th</sup> of September 2011). (In Spanish).
- [4] Corrección de errores del Real Decreto 1467/2007, de 2 de noviembre por el que se establece la estructura del bachillerato y se fijan sus enseñanzas mínimas. (Corrigendum Law 1467/2007 of 2 November for establishing the structure of Spanish high school and set their minimum education) Available at <http://www.educacion.es/educacion/que-estudiar/bachillerato.html> (Accessed on 12<sup>th</sup> of September 2011). (In Spanish).
- [5] N.A. Christakis, J.H. Fowler, *Connected: The Surprising Power Of Our Social Networks And How They Shape Our Lives*, Brown and Company. Hachette Book Group, 2009.
- [6] N.A. Christakis, J.H. Fowler, The spread of obesity in a large social network over 32 years, *The New England Journal of Medicine*, 357 (2007) 370–379.
- [7] M. Fernández Enguita, L. Mena Martínez, J. Riviere Gómez, *Fracaso y Abandono Escolar en España (Academic Underachievement and School Abandon in Spain)*, Colec. Estudios Sociales, Vol. 29, Fundación La Caixa, 2010. (In Spanish).
- [8] <http://www.educacion.es/mecd/jsp/plantilla.jjsp?id=310&area=estadisticas> (Accessed on 12<sup>th</sup> of September 2011). (In Spanish).
- [9] <http://www.educacion.es/mecd/jsp/plantilla.jjsp?id=3131&area=estadisticas> (Accessed on 12<sup>th</sup> of September 2011).
- [10] J.D. Murray, *Mathematical Biology*, Springer, 2002.
- [11] C. Fierro–Hernández, Patrón de rasgos personales y comportamiento escolar en jóvenes (Personal trait patterns and scholar behaviour), *Revista de Educación*, N.332, (2000) 291–304. (In Spanish).
- [12] <http://www.wolfram.com/products/mathematica>.
- [13] F.J. Santonja, A. Morales, R.J. Villanueva, J.C. Cortés, Analysing the effect of public health campaigns on reducing excess weight: A modelling approach for the Spanish Autonomous Region of the Community of Valencia, *Evaluation Program Planning* 35 (2012) 34–39.