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# THE $N^{*}(1520) \rightarrow \Delta \pi$ AMPLITUDES EXTRACTED FROM THE $\gamma p \rightarrow \pi^{+} \pi^{-} p$ REACTION AND COMPARISON TO QUARK MODELS 

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#### Abstract

The $\gamma p \rightarrow \pi^{+} \pi^{-} p$ reaction, in combination with data from the $\pi N \rightarrow \pi \pi N$ reaction, allows one to obtain the $s$ - and $d$-wave amplitudes for the $N^{*}(1520)$ decay into $\Delta \pi$ with absolute sign with respect to the $N^{*}(1520) \rightarrow N \gamma$ helicity amplitudes. In addition one obtains the novel information on the $q$ dependence of the amplitudes. This dependence fits exactly with the predictions of the non-relativistic constituent quark models. The absolute values provided by these models agree only qualitatively, and a discussion is done on the reasons for it and possible ways to improve.


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A recent detailed study of the $\gamma p \rightarrow \pi^{+} \pi^{-} p$ reaction [1], improving on the model of Lüke and Söding [2], together with a new experiment [3, 4], has stressed the role of the $N^{*}(1520)$ resonance which is essential to understand the total cross section for the $\gamma p \rightarrow \pi^{+} \pi^{-} p$ reaction for photon energies around $E_{\gamma}=700 \mathrm{MeV}$.

In ref. [1] it is shown that the peak observed in the total cross section for the $\gamma p \rightarrow \pi^{+} \pi^{-} p$ reaction around $E_{\gamma}=700 \mathrm{MeV}$ [3, (6, 5, 6] is due to the interference of the dominant term of the reaction, the contact gauge term $\gamma N \rightarrow \Delta \pi$ (the $\Delta$-Kroll-Ruderman term) and the $\gamma N \rightarrow N^{*}(1520) \rightarrow \Delta \pi$ process, when the decay of the $N^{*}(1520)$ into $\Delta \pi$ is through the $s$-wave. More recently, these results have been confirmed in ref. [7] where a simplified model with respect to the one in ref. [1] is used for different isospin channels of the $\gamma p \rightarrow \pi \pi N$ reaction.

In this paper we show how we can obtain the amplitudes for the $N^{*}(1520) \rightarrow$ $\Delta \pi$ process from the $\gamma p \rightarrow \pi^{+} \pi^{-} p$ reaction and their momentum dependence, which provides a nice test for the quark models.

The first ingredient in the $\gamma N \rightarrow N^{*}(1520) \rightarrow \Delta \pi$ process is the $N^{*}(1520) N \gamma$ coupling, which is given by [1]:

$$
\begin{equation*}
-i \delta H_{N^{\prime *} N \gamma}=i \tilde{g}_{\gamma} \vec{S} \cdot \vec{\varepsilon}+\tilde{g}_{\sigma}(\vec{\sigma} \times \vec{S}) \cdot \vec{\varepsilon} \tag{1}
\end{equation*}
$$

by means of which one reproduces the two helicity decay amplitudes. In Eq. (1]) $\vec{\sigma}$ are the ordinary spin Pauli matrices, $\vec{S}$ is the transition spin operator from $1 / 2$ to $3 / 2$ and $\vec{\epsilon}$ the photon polarization vector in the Coulomb Gauge. From the average experimental values of the helicity amplitudes given in [8] we get $\tilde{g}_{\gamma}=0.108$ and $\tilde{g}_{\sigma}=-0.049$.

For the $N^{*}(1520) \Delta \pi$ coupling, the simplest Lagrangian allowed by conservation laws is given by [1]:

$$
\begin{equation*}
\mathcal{L}_{N^{\prime *} \Delta \pi}=i \tilde{f}_{N^{\prime *} \Delta \pi} \bar{\Psi}_{N^{\prime *} \phi^{\lambda} T^{\lambda} \Psi_{\Delta}+\text { h.c. }} \tag{2}
\end{equation*}
$$

where $\Psi_{N^{\prime *}}, \phi^{\lambda}$ and $\Psi_{\Delta}$ stand for the $N^{*}(1520)$, pion and $\Delta(1232)$ field respectively, $T^{\lambda}$ is the $1 / 2$ to $3 / 2$ isospin transition operator.

However, such a Lagrangian only gives rise to $s$-wave $N^{*}(1520) \rightarrow \Delta \pi$ decay, while experimentally we know that there is a large fraction of decay into $d$-wave too [8, (9]. Furthermore, the amplitude of Eq. (22) provides a spin independent amplitude, while non relativistic constituent quark models (NRCQM) give a clear spin dependence in the amplitude. We propose here for this coupling the following Lagrangian, which, as we shall see, is supported by both the experiment and the NRCQM. The Lagrangian is given by

$$
\begin{equation*}
\mathcal{L}_{N^{\prime *} \Delta \pi}=i \bar{\Psi}_{N^{\prime * *}}\left(\tilde{f}_{N^{\prime *} \Delta \pi}-\frac{\tilde{g}_{N^{\prime *} \Delta \pi}}{\mu^{2}} S_{i}^{\dagger} \partial_{i} S_{j} \partial_{j}\right) \phi^{\lambda} T^{\lambda} \Psi_{\Delta}+\text { h.c. } \tag{3}
\end{equation*}
$$

with $\mu$ the pion mass.
This Lagrangian gives us the vertex contribution to the $N^{*}(1520)$ decay into $\Delta \pi$ :

$$
\begin{equation*}
-i \delta H_{N^{\prime *} \Delta \pi}=-\left(\tilde{f}_{N^{\prime *} \Delta \pi}+\frac{\tilde{g}_{N^{\prime *} \Delta \pi}}{\mu^{2}} \vec{S}^{\dagger} \cdot \vec{q} \vec{S} \cdot \vec{q}\right) T^{\lambda} \tag{4}
\end{equation*}
$$

where $\vec{q}$ is the pion momentum. In order to fit the coupling constants $\tilde{f}_{N^{\prime *} \Delta \pi}$ and $\tilde{g}_{N^{\prime *} \Delta \pi}$ to the experimental amplitudes in $s$ - and $d$-wave [8] we make a partial wave expansion [10] of the transition amplitude $N^{*}(1520)$ to $\Delta \pi$ from a state of spin $3 / 2$ and third component $M$, to a state of spin $3 / 2$ and third component $M^{\prime}$, following the standard "baryon-first" phase convention [11]:

$$
\begin{align*}
& -i\left\langle\frac{3}{2} M^{\prime}\right| \delta H_{N^{\prime *} \Delta \pi}\left|\frac{3}{2}, M\right\rangle=A_{s} Y_{0}^{M-M^{\prime}}(\theta, \phi)+ \\
& A_{d}\left\langle 2, \frac{3}{2}, M-M^{\prime}, M^{\prime} \mid 2, \frac{3}{2}, \frac{3}{2}, M\right\rangle Y_{2}^{M-M^{\prime}}(\theta, \phi) \tag{5}
\end{align*}
$$

where $\left\langle j_{1} j_{2} m_{1} m_{2} \mid j_{1} j_{2} J M\right\rangle$ is the corresponding Clebsch-Gordan coefficient, $Y_{l}^{m}(\theta, \phi)$ are the spherical harmonics, and $A_{s}$ and $A_{d}$ are the $s$ - and $d$-wave partial amplitudes for the $N^{*}(1520)$ decay into $\Delta(1232)$ and $\pi$, which are given by:

$$
\begin{align*}
& A_{s}=-\sqrt{4 \pi}\left(\tilde{f}_{N^{\prime *} \Delta \pi}+\frac{1}{3} \tilde{g}_{N^{\prime *} \Delta \pi} \frac{\vec{q}^{2}}{\mu^{2}}\right) \\
& A_{d}=\frac{\sqrt{4 \pi}}{3} \tilde{g}_{N^{\prime *} \Delta \pi} \frac{\vec{q}^{2}}{\mu^{2}} \tag{6}
\end{align*}
$$

From Eq. (5) we get the expression for the $N^{*}(1520)$ decay width into $\Delta \pi$ :

$$
\begin{equation*}
\Gamma=\frac{1}{4 \pi^{2}} \frac{m_{\Delta}}{m_{N^{\prime *}}} q\left(\left|A_{s}\right|^{2}+\left|A_{d}\right|^{2}\right) \theta\left(m_{N^{\prime *}}-m_{\Delta}-\mu\right) \tag{7}
\end{equation*}
$$

where $q$ is the momentum of the pion. We then fit the $s$ - and $d$-wave parts of $\Gamma$ to the average experimental values [8] by keeping the ratio $A_{s} / A_{d}$ positive as deduced from the experimental analysis of the $\pi N \rightarrow \pi \pi N$ reaction [9]. We get then two different solutions which differ only in a global sign,

$$
\begin{array}{lll}
\text { (a) } & \tilde{f}_{N^{\prime *} \Delta \pi}=0.911 &  \tag{8}\\
(b) & \tilde{g}_{N^{\prime *} \Delta \pi}=-0.552 \\
\tilde{f}_{N^{\prime *} \Delta \pi}=-0.911 & & \tilde{g}_{N^{\prime *} \Delta \pi}=0.552
\end{array}
$$

Now, the $\gamma p \rightarrow \pi^{+} \pi^{-} p$ reaction allows us to distinguish between both solutions, hence providing the relative sign with respect to the $N^{*}(1520) \rightarrow \gamma N$ amplitude.

In Fig. 1 we have plotted the total cross section for both solutions (solid lines). As we can see, only solution (a) fits the experiment, while the other one under-estimates the experimental cross section by a large amount. In Fig. 1 we also show the uncertainties in the cross section due to the experimental
errors in the $N^{*}(1520)$ helicity amplitudes and $s$ - and $d$-wave $\Delta \pi$ decay widths (region between dashed lines). These errors are calculated by evaluating the results a large number of times, $N$, with random values of the couplings within experimental errors. The deviation $\sigma$ from the mean, $\bar{x}$, is then obtained as (12):

$$
\begin{equation*}
\sigma^{2}=\frac{\sum_{i}\left(x_{i}-\bar{x}\right)^{2}}{N-1} \tag{9}
\end{equation*}
$$

For the width of the $N^{*}(1520)$ in the propagator we have taken the explicit decay into the dominant channels $(N \pi, \Delta \pi, N \rho)$ with their energy dependence, improving on the results of [1] where the energy dependence was associated to the $N \pi$ channel.

Because of the $N^{*}(1520)$ is a $d$-wave resonance, the energy dependence of the decay width into $N \pi$ is given by:

$$
\begin{equation*}
\Gamma_{N^{\prime *} \rightarrow N \pi}(\sqrt{s})=\Gamma_{N^{\prime *} \rightarrow N \pi}\left(m_{N^{\prime *}}\right) \frac{q_{c . m . m}^{5}(\sqrt{s})}{q_{c . m .}^{5}\left(m_{N^{\prime *}}\right)} \theta(\sqrt{s}-m-\mu) \tag{10}
\end{equation*}
$$

where $\Gamma_{N^{\prime *} \rightarrow N \pi}\left(m_{N^{\prime *}}\right)=66 \mathrm{MeV}$ [8], $q_{c . m .}\left(m_{N^{\prime *}}\right)=456 \mathrm{MeV}$ and $q_{c . m .}(\sqrt{s})$ is the momentum of the decay pion in the $N^{*}(1520)$ rest frame.

For the $\Delta \pi$ channel, the energy dependence of the decay width is given by Eq. (7).

Finally, for the $N^{*}(1520)$ decay into $N \pi \pi$ through the $N \rho$ channel is given by:

$$
\begin{equation*}
\Gamma_{N^{\prime *} \rightarrow N \rho[\pi \pi]}=\frac{m}{6(2 \pi)^{3}} \frac{m_{N^{\prime *}}}{\sqrt{s}} g_{\rho}^{2} f_{\rho}^{2} \int d \omega_{1} d \omega_{2}\left|D_{\rho}\left(q_{1}+q_{2}\right)\right|^{2}\left(\vec{q}_{1}-\vec{q}_{2}\right)^{2} \tag{11}
\end{equation*}
$$

where $q_{i}=\left(\omega_{i}, \vec{q}_{i}\right)(i=1,2)$ are the fourmomenta of the outgoing pions, $D_{\rho}\left(q_{1}+q_{2}\right)$ is the $\rho$ propagator including the $\rho$ width, $f_{\rho}$ is the $\rho \pi \pi$ coupling constant $\left(f_{\rho}=6.14\right)$, and $g_{\rho}$ is the $N^{\prime *} N \rho$ coupling constant ( $g_{\rho}=7.73$ ) that we fit from the experimental $N^{\prime *} \rightarrow N \rho[\pi \pi]$ decay width [8]. A slightly different, although equivalent treatment can be found in ref. [7].

The differences induced in the cross section from these improvements with respect to ref. [1] are, however, very small.

In Fig. 1 we are also plotting the experimental results of ref. [3] with the DAPHNE acceptance, together with our theoretical results with this acceptance (long dashed line). This is proper to do since the experimental total cross section is extrapolated from the measured one using the model of ref. [7].

We have also checked possible effects coming from off-shell effects in the propagators and vertices of the spin $3 / 2$ particles $\left(\Delta\right.$ and $N^{*}(1520)$ ) [13, [4]. By taking $A=-1$ and $Z \in[-1 / 2,1 / 2]$ the changes observed in the cross section are of the order of $1 \%$.

We should note that the interference between the $\gamma N \Delta \pi$-Kroll-Ruderman and the $\gamma N \rightarrow N^{*}(1520) \rightarrow \Delta \pi$ terms changes sign around $\sqrt{s}=m_{N^{\prime *}}$ where the real part of the $N^{*}(1520)$ propagator changes sign. This means that the on-shell value of the amplitudes $A_{s}$ and $A_{d}$ for the $N^{*}(1520) \rightarrow \Delta \pi$ decay plays no role at this energy and what matters is the value of $A_{s}$ (the one that interferes) at values of $q$ other than the one from the decay of the $N^{*}(1520)$ on-shell. This brings us to the $q$ dependence of the amplitude. While the $A_{d}$ part should have the $q^{2}$ dependence exhibited in Eq. (6)), the combination of $q^{2}$ which appears in $A_{s}$ is given by the chosen Lagrangian. One could, however, postulate other Lagrangians which would lead to a different combination. In order to investigate the most general $q^{2}$ dependence of $A_{s}$ we substitute $\tilde{f}_{N^{\prime *} \Delta \pi}$ by

$$
\begin{equation*}
\tilde{f}_{N^{\prime *} \Delta \pi}\left(1+\epsilon \frac{\vec{q}^{2}-\vec{q}_{o n-\text { shell }}^{2}}{\mu^{2}}\right) \tag{12}
\end{equation*}
$$

where $\vec{q}$ is the momentum of the decay pion, and $\vec{q}_{\text {on-shell }}^{2}$ is de momentum of the pion for a on-shell $N^{*}(1520)$ decaying into $\Delta \pi\left(\left|\vec{q}_{o n-\text { shell }}\right|=228 \mathrm{MeV}\right)$, and then we change $\epsilon$ comparing the results to the data. We find that, to a good approximation, $\epsilon=0$ gives the best agreement with the data, hence supporting the Lagrangian of Eq. (3).

In a recent paper [15] we use the information obtained here, together with all the other needed effective Lagrangians, in order to study the $\gamma N \rightarrow \pi \pi N$ reaction in all the isospin channels.

Next we pass to see what the NRCQM have to say with respect to this novel information. We followed a model designed by Bhaduri et al. [16] to describe the mesonic spectrum and which was used later on by Silvestre-Brac et al. [17] in the baryonic sector. The model has as starting point the quark-quark ( $q q$ ) potential

$$
\begin{equation*}
V_{q q}=\frac{1}{2} \sum_{i<j}\left(-\frac{\kappa}{r_{i j}}+\frac{r_{i j}}{a^{2}}+\frac{\kappa_{\sigma}}{m_{i} m_{j}} \frac{\exp \left(-r_{i j} / r_{0}\right)}{r_{0}^{2} r_{i j}} \overrightarrow{\sigma_{i}} \cdot \overrightarrow{\sigma_{j}}-D\right) \tag{13}
\end{equation*}
$$

incorporating the basic QCD motivated confining, coulombic and spin-spin $q q$ interactions, where the parameters are chosen in order to reproduce the low energy baryonic spectrum.

In order to study strong pionic decays $B \rightarrow B^{\prime} \pi$ we shall follow the elementary emission model (EEM) in which the decay takes place through the emission of a (point-like) pion by one of the quark. Some choices for the $q q \pi$ Hamiltonian are possible. We quote here a pseudovector interaction

$$
\begin{equation*}
H_{q q \pi}=\frac{f_{q q \pi}}{\mu} \bar{\Psi}_{q}(x) \gamma^{\nu} \gamma_{5} \vec{\tau} \Psi_{q}(x) \partial_{\nu} \vec{\phi}(x) \tag{14}
\end{equation*}
$$

The non-relativistic approach comes from the non-relativistic expansion of Eq. (14) in powers of $\left(p / m_{q}\right)$, where $p$ is the quark momentum operator. Up to first order in $\left(p / m_{q}\right)$ the Hamiltonian governing the transition $B \rightarrow B^{\prime} \pi^{\alpha}$ has this form:

$$
\begin{equation*}
H_{q q \pi} \propto f_{q q \pi}\left(\tau_{\alpha}\right)^{\dagger}\left[\vec{\sigma} \cdot \vec{q} e^{-i \vec{q} \vec{r}}-\frac{\omega_{\pi}}{2 m_{q}} \vec{\sigma} \cdot\left(\vec{p} e^{-i \vec{q} \vec{r}}+e^{-i \vec{q} \vec{r}} \vec{p}\right)\right] \tag{15}
\end{equation*}
$$

The isospin $(\vec{\tau})$, spin $(\vec{\sigma})$ and momentum $(\vec{p})$ operators stand for the quark responsible for the emission, and $\omega_{\pi}, \vec{q}$ are the energy and momentum of the emitted pion respectively. In Eq. (15) one distinguishes the term proportional to $\vec{q}$ (direct term) and the recoil term with the $\vec{p}$ structure.

There are two independent helicity amplitudes for the $N^{*}(1520) \rightarrow \Delta \pi$ decay. If we take the quantization axis along the pion momentum in the resonance rest frame, the helicity amplitudes correspond to a resonance spin projection, and we denote them as $A_{1 / 2}$ and $A_{3 / 2}$. After performing the calculations the ratio between them is:

$$
\begin{equation*}
\frac{A_{3 / 2}}{A_{1 / 2}}=\frac{-C_{\mathrm{REC}}}{C_{\mathrm{DIR}}-C_{\mathrm{REC}}} \tag{16}
\end{equation*}
$$

where $C_{\mathrm{DIR}}(\mathrm{REC})$ is the contribution from the direct (recoil) term. Rigorously, what we call $C_{\text {DIR }}$ contains a small piece, proportional to $\frac{\omega_{\pi}}{6 m_{q}}$ coming from the recoil term in (15).

The amplitudes $A_{s}$ and $A_{d}$ of Eq. (66) can be expressed in terms of these helicity amplitudes as follows 18:

$$
\begin{align*}
& A_{d} \propto A_{3 / 2}-A_{1 / 2} \\
& A_{s} \propto A_{3 / 2}+A_{1 / 2} \tag{17}
\end{align*}
$$

and their ratio in the EEM is given by

$$
\begin{equation*}
\frac{A_{d}}{A_{s}}=\frac{C_{\mathrm{DIR}}}{2 C_{\mathrm{REC}}-C_{\mathrm{DIR}}}=+0.156 \tag{18}
\end{equation*}
$$

where we have quoted the value obtained with the Bhaduri potential (13). The experimental value for this ratio is 1.2 [8].

Let us first discuss the sign. Notice that if only the direct term were present, the relative sign would be negative (the so-called $\mathrm{SU}(6)_{W}$ signs) (19. The introduction of the first order (recoil) contribution provokes a change of $\operatorname{sign}$ (the anti-SU(6) ${ }_{W}$ situation) in agreement with the experiments [9]. This fact was pointed out long ago by Le Yaouanc et al. [20] by using the ${ }^{3} P_{0}$ model (that could be regarded to some extent as a ( $p / m_{q}$ ) model). We have checked this sign with a wide variety of $q q$ potentials and with the ${ }^{3} P_{0}$ model
also, and the anti- $\mathrm{SU}(6)_{W}$ signs remain. Moreover, we have explored in the EEM with harmonic oscillator wave functions under which conditions are the $\mathrm{SU}(6)_{W}$ signs recovered. The answer is that the radius of the nucleon has to be larger than $\approx 1 \mathrm{fm}$. Certainly, spectroscopy does not support such a big quark core radius. Hence, as a quite model independent conclusion, we can say that the recoil term is crucial to explain the anti- $\mathrm{SU}(6)_{W}$ signs, and it is generally bigger than the direct term.

It is interesting to contrast the model prediction with the information on the $q$ dependence which our analysis of the experiment has provided for $A_{s}$ and $A_{d}$. From Eq. (6) we find

$$
\begin{align*}
& A_{s}+A_{d}=-\sqrt{4 \pi} \tilde{f}_{N^{\prime *} \Delta \pi} \\
& A_{d}=\frac{\sqrt{4 \pi}}{3} \tilde{g}_{N^{\prime *} \Delta \pi} \frac{\vec{q}^{2}}{\mu^{2}} \tag{19}
\end{align*}
$$

Equation (19) summarizes in a practical way the empirical $q$ dependence of the amplitudes. Now let us see what the NRCQM gives. Eqs. (19) can be recast in terms of the helicity amplitudes as

$$
\begin{align*}
& A_{3 / 2} \propto \tilde{f}_{N^{\prime *} \Delta \pi}  \tag{20}\\
& A_{3 / 2}-A_{1 / 2} \propto \tilde{g}_{N^{\prime *} \Delta \pi} \vec{q}^{2}
\end{align*}
$$

which in terms of Eq. (16), by means of the direct and recoil terms of Eq. (15), can be rewritten as

$$
\begin{align*}
& C_{\mathrm{REC}} \propto \tilde{f}_{N^{\prime *} \Delta \pi} \\
& C_{\mathrm{DIR}} \propto \tilde{g}_{N^{\prime *} \Delta \pi} \vec{q}^{2} \tag{21}
\end{align*}
$$

Now it is straightforward to see that this is indeed the case. The $N^{*}(1520) \rightarrow$ $\Delta \pi$ transition matrix element with the direct term of Eq. (15) requires the second term in the expansion of $e^{-i \vec{q} r}$, since $N^{*}(1520)$ contains a radial excitation with respect to the $\Delta(1232)$. Hence, the direct term is proportional to $\vec{q}^{2}$. On the other hand the recoil term gets the dominant contribution from the unity in the expansion of the exponential and hence it is momentum independent. Thus the quark model prediction for the $\vec{q}^{2}$ dependence of the amplitudes is in perfect agreement with experiment. However, the strength of the terms and their ratio is not well reproduced. This is not surprising in view that the recoil term appears to be bigger than the direct one in the ( $p / m_{q}$ ) expansion of Eq. (15). The values met here for $\left(p / m_{q}\right)$ are in average bigger than 1 and one should then expect limitations due to the nonrelativistic character of the model.

The purpose of the present paper is not to solve this interesting problem which has already caught attention of some groups [21, 22. Our purpose has been to show the novel experimental information about the $q$ dependence of the $s$ - and $d$-wave $\Delta \pi$ decay amplitudes of the $N^{*}(1520)$ extracted from the $\gamma p \rightarrow \pi^{+} \pi^{-} p$ reaction, and how it fits with the structure of NRCQM. It also gives in addition an absolute sign with respect to the $N^{*}(1520)$ helicity amplitudes which agrees with the NRCQM.

As for the need to introduce relativistic effects to get the appropriate strength of the $s$ - and $d$-wave ratio it seems quite obvious, and some results show that the ratio improves when this is done. The method of [21] is probably an indirect way of introducing relativistic effects by taking different factors in front of the two terms in Eq. (15) which are the then fit to a large set of data. In ref. [22] a ${ }^{3} P_{0}$ model with relativized hadronic wave functions is used and contrasted to a large set of hadronic decays of the baryon spectrum, and, concretely for the $N^{*}(1520)$, the $s$ - and $d$-wave $\Delta \pi$ decay ratio improves considerably without still being in agreement with experimental data. In Table I we show the results obtained with all these models. These results indicate the importance of the relativistic effects and the need for more work. Another possibility is explored in ref. 24 by making an expansion in powers of $(p / E)$ instead of $\left(p / m_{q}\right)$. However when trying to improve on this ratio it will be important to take into account the new experimental constraint obtained in the present work, and summarized in Eq. (19). While the second equation, establishing $A_{d}$ as a quadratic function of $q$, will come out relatively naturally in most schemes, the independence of $q$ of $A_{s}+A_{d}$ of the first equation is less than obvious and will pose a challenge to any new scheme.

In Fig. 1 we are also plotting the results obtained by using the strong and electromagnetic couplings for the $N^{*}(1520)$ resonance from the work of refs. [22, 23]. The results obtained are very close to those obtained with our model. This is so in spite that the individual electromagnetic and strong couplings are in some disagreement with experiment [8]. Indeed, the helicity amplitudes of [23] are smaller than experiment and the $s$-wave $N^{*}(1520) \rightarrow \Delta \pi$ amplitude of [22] (the relevant one in the interference) bigger tan the experiment, and there is a certain compensation of both deficiencies in the $\gamma p \rightarrow \pi^{+} \pi^{-} p$ cross section. This observation is interesting because it tells us that the fairness of a model for the $\gamma N \rightarrow \pi \pi N$ reaction is not enough by itself and one has to contrast the information provided by the model with the complementary experimental information extracted from the $\pi N \rightarrow \gamma N$ and the $\pi N \rightarrow \pi \pi N$ reactions. As an example in ref. 25] we show a model which gives equally good results as the present one in the $\gamma p \rightarrow \pi^{+} \pi^{-} p$ reaction and which has a ratio $A_{d} / A_{s}$ of opposite sign to the experimental one. These two examples show clearly the importance of using the information of several experiments in order to obtain the proper information on the properties of resonances, the
$N^{*}(1520)$ in particular in the present case.

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## Figure Captions

Fig. 1: Continuous line: Total cross section for the $\gamma p \rightarrow \pi^{+} \pi^{-} p$ reaction for different solutions of $\tilde{f}_{N^{\prime *} \Delta \pi}$ and $\tilde{g}_{N^{\prime *} \Delta \pi}$ (see Eq. (8)). Region between short-dashed lines: Uncertainties in the cross section due to the experimental errors in the $N^{*}(1520)$ helicity amplitudes and $s$ - and $d$-waves $\Delta \pi$ decay widths. Long-dashed line: Cross section integrated over the DAPHNE detector acceptance [3]. Dash-dotted lines: Total cross section with the Capstick et al. values of the strong and electromagnetic couplings [22, 23].

## Table Captions

Table 1: $\Gamma_{d} / \Gamma_{s}$ Ratio for different models. Experimental value from [B].

|  | EEM <br> with Eq. (15) | EEM <br> [21 | ${ }^{3} P_{0}$ <br> [22 | Exp. |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\Gamma_{d}}{\Gamma_{s}}$ | 0.024 | 0.139 | 0.069 | $1.4 \pm 0.6$ |



