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STOPPING OF CHARGED PARTICLES IN DENSE ONE-COMPONENT PLASMAS

In this paper, we examine the energy losses of charged particles, moving at different initial velocities in an electron fluid. It is illustrated that the stopping power at high velocities lies below the asymptotics of Bethe-Larkin. At low particle velocities, v , the dependence of energy losses on velocity in the random phase approximation behaves rectilinear. In the current article, we use the method of moments, which allows us to determine the stopping power of a non-ideal plasma without small-parameter expansion. The universality of this approach is that it allows one to use for calculations various effective potentials of interparticle interaction. Another important advantage of the approach is the opportunity to determine the dynamic characteristics of Coulomb systems by obtained static ones, that can be found from the solution of the Ornstein-Zernike equation in the hypernetted chain approximation, using the potentials specified in the work. The peculiarity of calculations in the method of moments application consists in the determination of so-called Nevanlinna parameter-function, included in the computed relations. In this contribution, we employ an empirical expression for Nevanlinna parameter-function.

Key words: one-component plasma, stopping power, method of moments, Coulomb system, Nevanlinna formula.

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Тығыз біркомпонентті плазмадағы зарядталған бөлшектердің тежелуі

Бұл жұмыста әртүрлі бастапқы жылдамдықтармен қозғалатын электронды сұйықтықтағы зарядталған бөлшектердің энергетикалық шығындары қарастырылады. Жоғары жылдамдықтағы тежелу қабілеті Бета-Ларкин асимптотикасынан төмен жатқаны дәлелденген. v бөлшектердің төменгі жылдамдықтарында v -дің энергетикалық шығынының тәуелділігі хаустық фазалардың жуықтауы жұмысында көрсетілгендей түзу сызықты болады. Осы ескертуде кіші параметр бойынша ыдырауды пайдаланбай, идеалды емес плазманың тежелу қабілетін анықтауға мүмкіндік беретін моменттер әдісі қолданылады. Берілген әдістің әмбебаптылығы бөлшекаралық өзара әсерлесудің есептеулері үшін әртүрлі эффе́ктивті потенциалдарын қолдануға мүмкіндік береді. Моменттер әдісін қолданып, есептеу жүргізудің ерекшелігі есептік қатынастарға түсетін Неванлинна параметр-функциясын анықтаудың қажеттілігі болып табылады. Осы мақалада бұрын ұсынылған қатынас қолданылды. Берілген әдістің маңызды артықшылығы жұмыста көрсетілген потенциалдар көмегімен гиперте́збекті жуықтаудағы Орнштейн-Церник теңдеуінің

шешімінен табылуы мүмкін болатын есептелген статикалық кулондық жүйелердің динамикалық сипаттамаларын анықтау болып табылады.

Түйін сөздер: біркомпонентті плазма, тежелу қабілеті, моменттер әдісі, кулондық жүйе, Неванлинна формуласы, шығындар функциясы.

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Торможение заряженных частиц в плотной однокомпонентной плазме

В данной работе рассматриваются энергетические потери заряженных частиц в электронной жидкости, движущихся с различными начальными скоростями. Показано, что тормозная способность на больших скоростях лежит ниже асимптотики Бета-Ларкина как было приведено в работах других авторов. Показано, что при малых скоростях v частиц зависимость энергетических потерь от v ведет себя прямолинейно, как это было показано ранее в с диэлектрической функцией в приближении хаотических фаз. В настоящей заметке используется метод моментов, который позволяет определять тормозную способность неидеальной плазмы, не используя разложения по малому параметру. Универсальность данного подхода позволяет использовать для расчетов различные эффективные потенциалы межчастичного взаимодействия. Особенностью вычислений с использованием метода моментов является необходимость определения так называемой параметр-функции Неванлинны, входящей в расчетные соотношения. В данной статье использовано соотношение, предложенное нами ранее. Важным достоинством данного подхода является возможность определения динамических характеристик кулоновских систем по рассчитанным статическим, которые могут быть найдены из решения уравнения Орнштейна-Цернике в гиперцепном приближении с помощью потенциалов указанных в работе.

Ключевые слова: однокомпонентная плазма, тормозная способность, метод моментов, кулоновская система, формула Неванлинны, функция потерь.

Introduction

One of the challenges of the statistical plasma physics is the description of the transition from collisionless to collision dominated regimes in different Coulomb systems, of the crossover from classical to Fermi liquid behavior in dense plasmas [1, 2]. We refer to strongly coupled plasmas characterized by a wide range of variation of temperature and mass density spanning a few orders of magnitude. Under such conditions thermal, Coulomb coupling, and quantum effects compete between them and impede the construction of a bridge theory capable of including of all of these effects in the description of static, kinetic, and dynamic properties of the above systems of high relevance for inertial fusion devices [3] and advanced laboratory studies, e.g., in ultracold plasmas [4], electrolytes and charged stabilized colloids [5], laser-cooled ions in cryogenic traps [6], and dusty plasmas [7]. In Nature strongly coupled

plasmas appear in various settings as well, e.g. in white dwarfs and neutron stars [8].

The scientific and technological revolution that began in the last century has greatly increased the energy needs of mankind [11]. This led the scientists to turn to one of the most promising areas in the energy sector – energy production using the reaction of controlled thermonuclear fusion due to the fusion of light nuclei with the consequent release of enormous amounts of energy. One of the problems that arise in connection with this problem is the heating of the plasma to high temperatures. If, initially, there were used powerful lasers [12, 13] for this purpose, recently beside that beams of charged ions [14] have also been used.

The experiments, related to the interaction of the plasma and the ion beam moving in it, stimulated the development of theoretical methods for determining the energy losses of a charged particle in a plasma medium, i.e. the study of the so-called stopping power of the plasma due to the polarizational losses.

Polarizational losses

In 1930, Bethe developed a formula for the energy loss by a fast particle, assuming that the atoms of the medium behave like quantum-mechanical oscillators [15]. Later, Larkin [16] had shown that in the case when fast ions penetrate the electron gas, a similar formula is applicable, but with the replacement of the average excitation frequency by the plasma frequency ω_p :

$$-\frac{dE}{dx} \underset{v \gg v_p}{\simeq} \left(\frac{Z_p e \omega_p}{v} \right)^2 \ln \frac{2mv^2}{\hbar \omega_p}, \quad (1)$$

where $Z_p e$ and v are the charge and velocity of particle, respectively.

The polarization mechanism is used to calculate the energy losses of a fast particle passing through the Coulomb system in disregard of the collision and ionization losses. In 1959, Lindhard received an expression, relating the energy loss due to polarization with the dielectric function of the medium [17]:

$$\frac{dE}{dx} = \frac{2(Z_p e)^2}{\pi v^2} \int_0^\infty \frac{dk}{k} \int_0^{kv} \omega \operatorname{Im} \varepsilon^{-1}(k, \omega) d\omega. \quad (2)$$

This ratio gives the relationship of polarization energy losses of a moving charged particle in a plasma with the longitudinal permittivity of the medium $\varepsilon(k, \omega)$. From his view, we can conclude that the loss of energy of the test charge in the plasma does not depend on the mass of particles and depends only on its charge and velocity.

Formula (2) for the calculating of the polarizational losses of the test charge, moving in the plasma, is valid in the one-particle approximation, in which the deceleration of the ion beam is represented as the deceleration of single ions not interacting with each other. This approximation is valid for ion flux densities of many smaller medium densities, which is performed for most modern experiments.

In this paper, we investigate the stopping power (2) of a one-component hydrogen Coulomb system for the dielectric function, found by the method of moments [18].

Plasma parameters

The potential of the interparticle interaction is

$$\varphi(r) = \frac{e^2}{r},$$

and to describe the state of the plasma, the parameters of coupling and density are respectively used

$$\Gamma = \frac{e^2}{ak_B T}, \quad r_s = \frac{a}{a_B}.$$

The Wigner-Seitz radius is entered here as

$$a = \sqrt[3]{3 / 4\pi n},$$

where e – electron charge, k_B – Boltzmann constant, T – temperature, a_B – Bohr radius, n – plasma concentration.

Method of moments

In order to study the properties of non-ideal plasma with the parameters of the coupling and degeneration of the order and more than unity, we should use the non-perturbative method of moments [18], which does not require any small parameters.

This method allows one to determine the dielectric properties of the Coulomb system, using the first few moments of the loss function,

$$L(k, \omega) = -\frac{1}{\omega} \operatorname{Im} \frac{1}{\varepsilon(\omega, k)}, \quad (3)$$

which can be calculated through the potential of interparticle interaction and the static structural factors of the system. The latter can be computed from the solution of the Ornstein-Zernike equation in the hypernetted chain approximation (HNC) [19].

We can write the Nevanlinna formula that determines the dielectric properties of the medium in the form of [18]:

$$\begin{aligned} \frac{1}{\varepsilon(k, \omega)} &= \\ &= 1 + \frac{\omega_p^2(\omega + Q(k))}{\omega(\omega^2 - \omega_2^2(k)) + Q(k)(\omega^2 + \omega_1^2(k))} \end{aligned} \quad (4)$$

Here $\omega_1^2(k) = C_2(k) / C_0(k)$, $\omega_2^2(k) = C_4(k) / C_2(k)$, and $Q(k) = \frac{i}{\sqrt{2}} \frac{\omega_2^2(k)}{\omega_1(k)}$ is the function of the Nevanlinna class, obtained in [20]. The parameters are defined as the power frequency moments of the loss function:

$$C_\nu(k) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \omega^{\nu-1} \text{Im} \varepsilon^{-1}(k, \omega) d\omega. \quad (5)$$

Evaluation of moments allows us to write explicit expressions for them, based on the following considerations. The zero moment is obtained using the quantum potential [21] by the dielectric response method [22]:

$$C_0(k) = \frac{\kappa^2 \chi^4}{q^4 \kappa^2 + q^2 \chi^4 + \kappa^2 \chi^4},$$

where

$$\kappa = \sqrt{6\Gamma}, \quad \chi = (12r_s)^{1/4}, \quad (6)$$

and $\beta^{-1} = k_B T$ is the temperature of system in energy units.

The second frequency moment of the loss function, according to the f -sum rule [18], is equal to the square of the plasma frequency of the system:

$$C_2 = \omega_p^2. \quad (7)$$

The expression for the fourth sum rule is written as follows

$$C_4(k) = \omega_p^4 (1 + K(k) + U(k)). \quad (8)$$

Below are the designations of the values included in (8)

$$K(k) = \frac{\langle v_{th} \rangle^2 k^2}{\omega_p^2} + \left(\frac{\hbar}{2m} \right)^2 \frac{k^2}{\omega_p^2}, \quad (9)$$

$$U(k) =$$

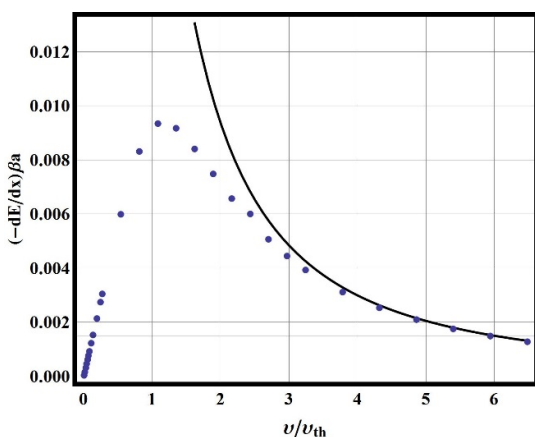
$$= \left(1 / 2\pi^2 n \right) \int_0^\infty p^2 [S(p) - 1] f(p, k) dp, \quad (10)$$

$\langle v_{th} \rangle^2$ is the square of average thermal velocity of electrons, m – their mass, \hbar – Plank’s constant, and

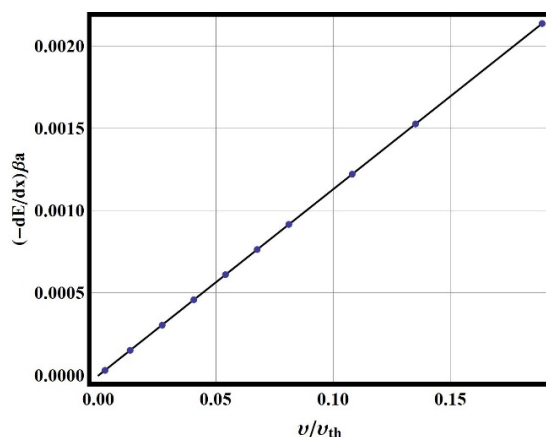
$$f(p, k) = 5/12 - (p^2 / 4k^2) + \frac{(k^2 - p^2)}{8pk^3} \ln \left| \frac{p+k}{p-k} \right|.$$

Obtained results

Figures 1-3 present the results of calculations of polarizational losses of heavy charged particles in a one-component plasma in a wide range of velocities (a) and particularly at low velocities (b).



Dots are calculated data by formula (2), the solid line is the asymptotic form of the Bethe-Larkin (1)



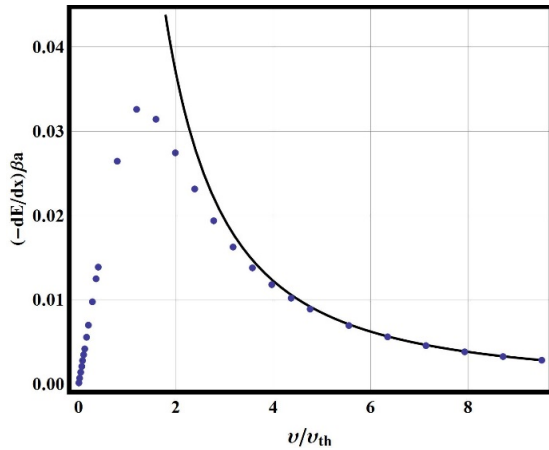
Dots are calculated data by formula (2), a solid line is a

straight line $C \frac{v}{v_{th}}$, where $C = 0.003$

(a)

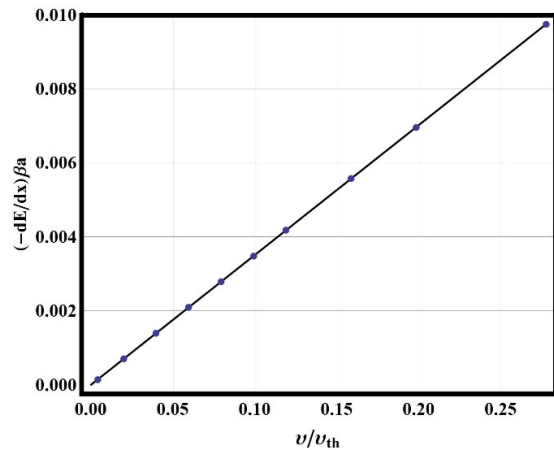
(b)

Figure 1 – Stopping power at $\Gamma = 0.05$ and $r_s = 2.5256$



Dots are calculated data by formula (2), the solid line is the asymptotic form of the Bethe-Larkin (1)

(a)

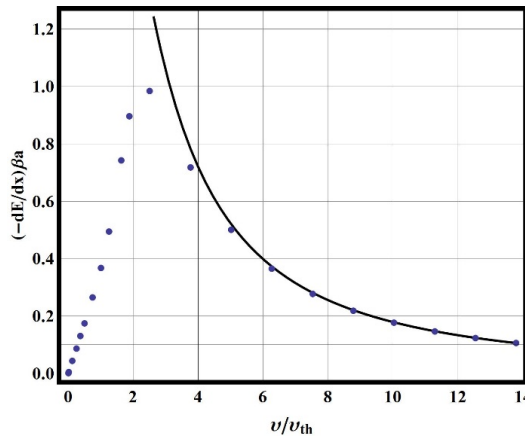


Dots are calculated data by formula (2), a solid line is a

straight line $C \frac{v}{v_{th}}$, where $C = 0.014$

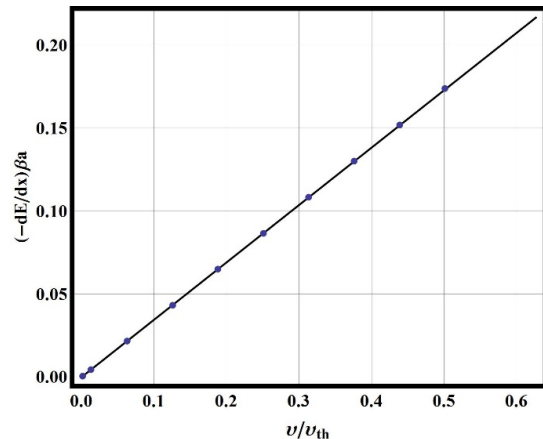
(b)

Figure 2 – Stopping power at $\Gamma = 0.11$ and $r_s = 2.5256$



Dots are calculated data by formula (2), the solid line is the asymptotic form of the Bethe-Larkin (1)

(a)



Dots are calculated data by formula (2), a solid line is a

straight line $C \frac{v}{v_{th}}$, where $C = 0.433$

(b)

Figure 3 – Stopping power at $\Gamma = 1.1$ and $r_s = 2.5256$

From the above plots that describe the stopping power of one-component plasma in the wide interval of the coupling parameter, it is possible to conclude that the good agreement of calculation results with

the asymptotics of the Bethe-Larkin, and for small velocities the curves are in good agreement with theoretical calculations, obtained with using of the ideas of [23].

Conclusion

The corresponding evaluated data in the part indicated by the letter (a), which represents the dependences of energy losses in one-component plasma on the velocity of projectiles, are always below the asymptotic Bethe-Larkin curve (1) just as it should be [24, 25]. At low velocities, the dependence of the stopping power (b) represents a

straight line. The last statement in the random phase approximation was proved in [23].

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